Functional Mathematics

For Junior High Schools

Teacher's Guide

Basic 8

Benjamin A. Worname

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PREFACE

Functional Mathematics for Junior High Schools, Basic 7 to 9 is a set of three books that have been carefully developed for use by learners in Junior High Schools. These books conform, by all standards to the new Mathematics Curriculum developed by the National Council for Curriculum and Assessment (NaCCA) September, 2020 for the Common Core Programme.

The books are written to assist learners develop the core skills namely:

- Critical Thinking and Problem Solving Abilities.
- Collaborative and Team Work.
- Personal Development and Leadership.
- Attention to Precision.
- Creativity and Innovation.
- Cultural Identity and Global Citizenship.
- Digital Literacy

Signigicant features of the three set of books include:

- The use of simple language and expressions with enough practical activities.
- The use of locally available materials as Teaching and Learning Resources (TLRs).
- Adequate relevant illustrations for easy understanding of the various concepts.
- Enough examples and exercises which could be useful for School Based Assessment (S.B.A)

The guides fulfill the requirement considering the in-depth treatment of the strands in the Mathematics curriculum. In addition, the guides provide activities, diagnostic assessment, progressive assessment and aditional infomation that further enhance both the facilitator's and learner understanding.

It is our hope that the Teacher's Guide and the Learner's Books would meet the needs of both facilitators and learners and help them to translate classroom interactions into effective teaching and learning. Facilitators must always consult the Teacher's Guides when using the Learner's Books so as to conform to the requirement of the Mathematics curriculum.

GENERAL AIM

The general aim of the curriculum is to develop individuals to become mathematically literate. Good problem solvers, have the ability to think creatively and possess the confidence and competence to participate fully in the affairs of the Ghanaian society as a responsible local and global citizen.

RATIONALE

The rationale is that, mathematics forms an integral part of our everyday lives and that development is hinged on mathematics. To provide quality Mathematics education, the teacher must facilitate learning of mathematics in the classroom. This will provide foundations for discovery and understanding the world around us and lay the grounds for mathematics and mathematics-related studies at higher levels of education.

TEACHING PHILOSOPHY

The teaching philosophy is that Ghana believes that an effective Mathematics education needed for sustainable development should be inquiry-based. Thus, mathematics education must provide learners with opportunities to expand, change, enhance and modify the ways in which they view the world. It should be pivoted on learner centred teaching and learning approaches that engage learners physically and cognitively in the knowledge-acquiring process in a rich and rigorous inquiry-driven environment.

LEARNING PHILOSOPHY

The learning philosophy is that mathematics learning is an active contextualised process of constructing knowledge based on learners' experiences rather than they acquiring new ones. Teachers serve as facilitators by providing the enabling environment that promote the construction of learners` own knowledge based on their previous experiences. Learners are information constructors who operate as researchers. This makes learning more relevant to the learners and leads to the development of critical thinkers and problem solvers.

LEARNING AND TEACHING APPROACHES

The core competencies describe the relevant global skills for learning that helps learners to develop in addition to arithmetic, writing, reading and creativity. The global skills for learning allow learners to become critical thinkers, problem-solvers, creators, good communicators, collaborators, digitally literate and culturally and globally sensitive citizens who are life-long learners with a keen interest in their personal development.

Pedagogical approaches; The common core programme (CCP) emphasises creative and inclusive pedagogies that are anchored on authentic and enquiring-based learning, collaborative and cooperative learning, differentiated learning and holistic learning as well as cross disciplinary learning.

SPECIFIC AIMS

The Mathematics curriculum is designed to help learners to achieve the following:

1. recognise that Mathematics permeates the world around us.
2. appreciate the usefulness, power and beauty of Mathematics.
3. enjoy mathematics and develop patience and persistence when solving problems.
4. understand and be able to use the language, symbols and notations of Mathematics.
5. develop Mathematical curiosity and use inductive and deductive reasoning when solving problems.
6. become confident in using Mathematics to analyse and solve problems both in school and in real-life situations.
7. develop the knowledge, skills and attitudes necessary to pursue further studies in Mathematics.
8. develop abstract, logical and critical thinking abilities to reflect critically upon their work and the works of others.

The Teacher's Guide continues explanation on some of the concepts and methodologies to be used by the teacher in teaching the learners in the classroom. The teacher`s guide is made up of solved questions on strands and sub-strand/units. It also provides answers to exercises in the Learner's Book as well as appropriate references to the Learner's Book. The teacher is expected to follow carefully all the strands, sub-strand/units and examples in order to achieve the following core competencies.

CORE COMPETENCIES

1. **Critical thinking and Problem Solving**
   Developing learners' cognitive and reasoning abilities to enable them analyse issues and situations leading to the resolution of problems. This skill enables learners to draw on and discuss what they have learned and from their own experiences.

2. **Creativity and Innovation**
   This competency promotes in learners, entrepreneurial skills through their ability to think of new ways of solving problems and developing technologies for addressing problems at hand.

3. **Communication and Collaboration**
   This competency promotes in learners, skills in making use of language, symbols and texts to exchange information about themselves and their life experiences. Learners actively participate in sharing their ideas, engage in dialogue with others by listening to and learning from them in ways that respect and value the many perspectives of all persons involved.

4. **Cultural Identity and Global Citizenship**
   Developing learners who put country and service foremost through an understanding of what it means to be active citizens. This is by inculcating in them a strong sense of

social and economic development awareness. Learners make use of the knowledge, skills and attitudes acquired to contribute effectively towards the socio-economic development of the country and on the global stage.

5. **Personal Development and Leadership**
Improving self-awareness, skills, building and renewing self-esteem, identifying and developing talents, fulfilling dreams and aspirations, learning from the mistakes and failures of the past and developing other people or meeting other people's needs. It involves the recognition of values such as honesty and empathy, seeking the well-being of others, distinguishing between right and wrong, fostering perseverance, resilience and self-confidence and developing love for lifelong learning.

6. **Digital Literacy**
Developing learners to discover, acquire, use and communicate through Information and Communication Technology (I.C.T) to support their learning and to make use of digital media responsibly.

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STRAND/CHAPTER 1: NUMBER

Sub-Strand/Unit 1: Number And Numeration Systems

Refer to Learner's Book 8, pages 1 - 61

Content Standard:
B8.1.1.1: Demonstrate understanding and the use of place value for expressing quantities in standard form and rounding numbers and decimals to significant figures and a given number of decimal places.

B8.1.1.2: Apply the concepts and vocabulary of sets on sets of factors of numbers to identify perfect squares, determine their square root and solve real life problems involving union and intersection of two sets.

Learning Expectations:
After studying this sub-strand/unit, the learner will be able to:
- skip count forward and backward.
- compare and order whole numbers using " > "," < "*and* " = "
- express integers of any size in standard form.
- express integers to a given number of significant figures and decimal places.
- identify perfect squares and determine the square root.
- solve real life problems involving union and intersection.

Keywords: *Refer to Learner's Book, Page 1*
Lead learners to use their dictionaries or the internet to find the contextual meaning of the keywords.
- *place value, skip count, greater than, less than, equal to, standard form, integers, significant figures, decimal place, perfect square, perfect numbers, square root, union, intersection*

Teaching and Learning Resources (TLRs):
A place-value chart, a perfect square, 1 to 30 chart.

Core Competencies
- Communication and Collaboration
- Critical Thinking and Problem Solving
- Creativity and Innovation

Lesson Presentation:
Revise learners' previous knowledge on place value and how to count forwards and backwards in 50s, 10s, and 100s.

Refer to pages 2-6 of Learner's Book 8.
Lead, guide and involve learners to solve the examples and exercises at these pages of the Learner's Book.

Skip count forward in 10,000s
Skip counting is a mathematical technique taught as a kind of multiplication in reform mathematics textbooks. Skip counting is a method of counting numbers by adding a number every time to the previous number. For example, skip counting by 2s means that, we start from 2 and add 2 to successive numbers. Thus 2,4,6,8 etc.
Now, to count forward in 10,000 , we have , 10,000 , 20,000, 30,000, 40,000, etc.

It can be seen that 10,000 is being added to each number going forward from left to right. This is what is referred to as count forward. And this can be done by any interval. Refer to page 5 of Learner's Book 8.

- Lead and guide learners through the examples at pages 5 – 6 of Learner's Book 8.
- Skip count backwards : To skip count backwards, we subtract from the first number to obtain the second number.

Refer to page 6 of Learner's Book 8.

Compare and order whole numbers using the " $>$ "," $<$ "*and* " $=$ " **signs**
Interpret the symbols to learners:

- Greater than, $>$
- Less than, $<$
- Equal to, $=$

Refer to page 7 of Learner's Book 8 and guide learners to read and discuss these symbols " $>$ "," $<$ "*and* " $=$ " among themselves.

To compare two numbers, we are looking at which one of the two numbers is bigger than the other or smaller than the other or whether the two numbers are the same.

For example: Compare 55 and 35
Here, 55 is more than 35, therefore $55 > 35$. Again, 35 is smaller than 55, hence $55 < 35$.

Note that $55 > 35$ and $55 < 35$ means the same thing in terms of interpretation. *Refer to page 8 of Learner's Book 8* and guide learners through the examples.
Let learners do the exercises.

Multiplying by Powers of 10 *(Refer to page 9 of Learner's Book 8)*
Lead and guide learners to write powers of 10. These can be written in two forms.

1. Writing powers of 10 with an exponent or index. For instance; $10^2, 10^3, 10^4, 10^5$, etc.
2. Writing in expanded form. For instance; 100, 1000, 10000, 100000, etc.

Note that, the number of zeros is equal to the exponent/index.
Refer to page 10 of Learner's Book 8 and guide learners to do example 1.

Dividing by powers of 10 *(Refer to pages 10 – 11 of Learner's Book 8)*
Lead learners to solve examples 1, 2 and 3.
Give further, examples for learners to do and let them discuss their solutions.

Standard Form *(Refer to page 11-13 of Learner's Book 8)*
Standard form or scientific notation is a method used to write very large numbers and very small numbers in a more concise form, retaining the significant digits, but without writing so many zeros or the insignificant digits.

Diagnostic Assessment

1. Write 483424 in standard form.

Standard form is written in the form $\boxed{A \times 10^n}$ where 'A' is a single non-zero digit, base 10 and 'n' is the exponent indicating the number of decimal place(s). It can be positive or negative depending on the number given to be written in standard form.

$$483424 = 4.83424 \times 10^5$$

We have an exponent of 5 which means, we moved the decimal point from right to left.
Refer to pages 12 – 14 of Learner's Book 8. Involve learners to solve other examples.

Example : Write 0.0000083424 in standard form.

$$0.00000083424 = 8.3424 \times 10^{-6}$$

The exponent is negative six because, we moved the decimal point six times from the left to right at the first non-zero digit of the number given. *Refer to pages 13 – 14* of Learner's Book 8.

Changing from standard form *(Refer to pages 14-16 of Learner's Book 8)*
If a number is given in standard form we can change it by writing it in normal form. In this case;

i. Move the decimal point to the right by the number of times as the exponent. For instance, 8.342×10^3
   We move the decimal point three places to the right since the exponent is three.

$$8.342 \times 10^3 = 8342$$

ii. We move the decimal point to the left if the exponent is a negative number. For instance, 7.568×10^{-3}

$$7.568 \times 10^{-3} = 0.007568$$

Put learners in groups to solve the questions at page 16 and let them discuss their solutions.

Significant figures *(Refer to pages 16-18 of Learner's Book 8)*
The significant figures of a number refer to those digits that have meaning in reference to a measured or specific value.

Lesson Presentation:
Guide and explain the following rules to learners.

Rules to follow in writing significant figures.

1. Non-zero single digits (1-9) and zeros, that are in between two non-zero digits are always significant. E.g., 20501, 3033, 2001, 100709.
2. Leading zeros are never significant. For instance; 06 0.0075, 017 etc.
3. Training zeros are only significant if a decimal point is present in the number. For instance;
   - 7000 has only one significant figure.
   - 0.200 has three significant figures.
   - 11.7035 has six significant figures.

Lead and guide learners to do the examples given at these pages. Give further examples to learners to do.

Express Figures in Decimal Places *(Refer to pages 18-20 of Learner's Book 8)*
Decimal places are the number of digits that appear after the decimal point. Example:

i. 2.345 has three decimal places
ii. 0.267 has three decimal places
iii. 231.00 has two decimal places
iv. 9.0 has one decimal place

Procedure to determine the number of decimal places
Lead and explain the steps involved to determine the number of stated decimal places to learners.

How many decimal places has;

i. 456.2005 Four decimal places
ii. 0.00796 Five decimal places
iii. 7.110 Three decimal places

Let learners do the example under application.
Refer to page 21 of Learner's Books.

SETS

Refer to pages 21-22 of Learner's Book 8.
A set is a collection of well-defined objects. We use capital letters (upper case) to represent a set or group of objects.

For instance, a set of natural numbers less than 11 can be represented as;

$$A = \{natural\ umbers\ less\ than 11\}$$

We can list the members or elements that are in set A

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Square Number
A number multiplied by itself gives us the square of that number.

- $2 \times 2 = 4$, therefore 4 is a square number
- $3 \times 3 = 9$, therefore 9 is a square number
- $10 \times 10 = 100$, therefore 100 is a square number

We can write the above examples as , $2^2 = 4, 3^2 = 9, 10^2 = 100$
Hence 4, 9 and 100 are square numbers.
Let learners write ten different square numbers and discuss them among their colleagues.

Perfect Square Numbers *(Refer to pages 22-24 of Learner's Book 8)*
A perfect square number is a number that can be expressed as the product of an integer itself or as the second exponent of an integer.

Examples are; 25,36,16,4,9, etc.

$$25 = 5 \times 5 = 5^2$$
$$36 = 6 \times 6 = 6^2$$
$$16 = 4 \times 4 = 4^2$$
$$9 = 3 \times 3 = 3^2$$

Perfect Numbers *(Refer to page 24 of Learner's Book 8)*
A perfect number is a positive integer that is equal to the sum of its factors except for the number itself.

Lesson presentation
Let learners provide the factors of certain numbers.

For example; Factors of 6,8,12,25, etc.

$$6 = \{1, 2, 3, 6\}$$

Find the sum of the factors listed except 6

$$1 + 2 + 3 = 6$$

This means that 6 is a perfect number.

Factors of $8 = \{1, 2, 4, 8\}$
Find the sum of the factors except 8

$$1 + 2 + 4 = 7$$

Since $8 \neq 7$, we say that 8 is not a perfect number.

Factors of $12 = \{1, 2, 3, 4, 6, 12\}$
Find the sum of the factors except 12

$$1 + 2 + 3 + 4 + 6 = 16$$

$12 \neq 16$, therefore 12 is not a perfect number.

Factors of $25 = \{1, 5, 25\}$
Find the sum of the factors except 25

$$1 + 5 = 6$$

$25 \neq 6$, therefore 25 is not a perfect number.

The Square Root of a Perfect Number *(Refer to page 24-24 of Learner's Book 8)*
The square root of a perfect number is equal to a number which when squared gives the original number.

For instance, $\sqrt{4} = 2$, since $2 \times 2 = 4$

$$\sqrt{49} = 7, \text{ since } 7 \times 7 = 49$$

$$\sqrt{36} = 6, \text{ because } 6 \times 6 = 36$$

Refer to page 25 of Learner's Book 8.

Prime Factorisation Method *(Refer to page 25 of Learner's Book 8)*
This is a strategy that can be used to find the square root of a number.

For example, the prime factors of 4

1. $4 = 2 \times 2$
   Hence, $\sqrt{4} = 2$

2. $169 = 13 \times 13$
   $\sqrt{169} = 13$
   $8 = 2 \times 2 \times 2$
   $= 4 \times 2$
   $= \sqrt{4} \times \sqrt{2}$
   $= 2\sqrt{2}$

8 is not a perfect square number, hence we leave the square root in surd form. Surds will be studied later.

Repeated Subtraction Method *(Refer to page 25-26 of Learner's Book 8)*
Lead and guide learners to work through the example at page 27 of Learner's Book 8.

Set of Factors *(Refer to page 28 of Learner's Book 8)*
A factor is a divisor of an integer or is an integer that may be multiplied by an integer to produce the number.

For instance, $2 \times 3 = 6$ means that 2 and 3 are factors of 6. Also $3 \times 4 = 12$, $2 \times 6 = 12$ shows that 2,3,4 and 6 are factors of 12.

During the lesson presentation, let learners give factors of certain numbers.

$$6 = \{1, 2, 3, 6\}$$
$$12 = \{1, 2, 3, 4, 6, 12\}$$
$$18 = \{1, 2, 3, 6, 9, 18\}$$

Put learners into groups of five and let them write the factors of different numbers and then present them to the class.

Set of Common Factors *(Refer to page 29 of Learner's Book 8)*
Finding common factors is the strategy that is used to determine elements or members that are common to two or more sets.

Lesson Presentation
Let learners list the factors of a particular number and also the factors of another number.
For example, let learners find the common factors of 12 and 18.

Let us list the factors of;

$$12 = \{1, 2, 3, 4, 6, 12\}$$
$$18 = \{1, 2, 3, 6, 9, 18\}$$

Common Factors

$$C.F = \{1, 2, 3, 6\}$$

It should be noted that the common factors form what is called the intersection of the two sets.

Set Notation *(Refer to page 30-31 of Learner's Book 8)*
Lead and guide learners to solve the examples at page 31 of the Learner's Book 8.

Diagnostic Assessment

1. Given that , $A = \{2, 3, 4, 5, 6, 7\} B = \{0, 3, 6, 9, 12\}$ and $C = \{2, 4, 6, 8\}$

   Find $A \cup B \cup C$

   Solution

   $$A \cup B \cup C = \{0, 2, 3, 4, 5, 6, 7, 8, 9, 12\}$$

Note that elements that repeat must be listed only once in the main set.

Intersection of Sets (∩) *(Refer to page 32- 33 of Learner's Book)*
Intersection of sets is the list of elements or members that are common to the given sets.

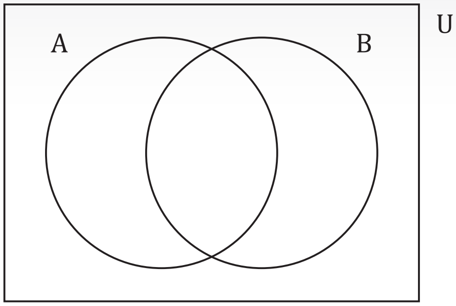
The symbol for intersection is " ∩ ".
Lead and guide learners to understand that;
1. If two sets A and B intersect, it is written as "$A \cap B$"
2. If three sets A, B and C intersect, then we have "$A \cap B \cap C$".

Give further examples of sets with common elements and let learners find their intersection.

Refer to page 33 of Learner's Book 8

Venn Diagram *(Refer to pages 34 - 42 of Learner's Book 8)*

Venn diagrams are pictorial representations of sets represented by closed figures. Venn Diagrams are used to illustrate various operations like; union, intersection and differences.

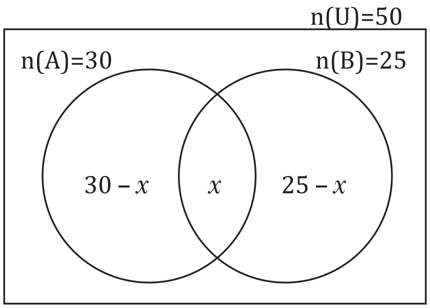


Guide learners to solve two-set problems

$$30 - x + x + 25 - x = 50$$
$$30 + 25 - x + x - x = 50$$
$$55 - x = 50$$
$$55 - 50 = x$$
$$5 = x$$
$$\therefore x = 5$$

Lead and guide learners to name and identify the various regions in a two-set Venn diagram by shading those regions respectively. *(Refer to page 37 of Learner's Book 8)*.

Guide learners to solve two-set problems.



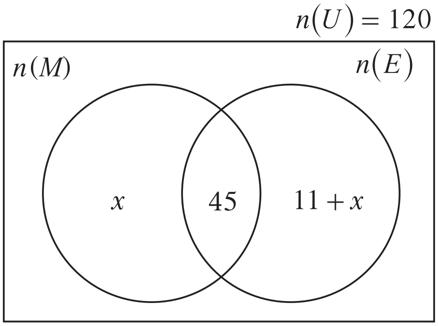
Diagnostic Assessment

In a school, 120 students passed in either Mathematics or English or both of these . 45 passed in both subjects and 11 more passed in English than in Mathematics.

i. Illustrate the information on a Venn diagram.
ii. How many students passed in Mathematics only?
iii. How many students passed English?

Solution

Total number of students $n(\cup) = 120$,
Number that passed both subjects; $n(M \cap E) = 45$
11 more passed in English than in Mathematics means, if x students passed Mathematics then $(11 + x)$ students passed in English.



i. $x + 45 + 11 + x = 120$

$56 + 2x = 120$

$2x = 120 - 56$

$2x = 64$

$x = 32$

ii. Those who passed Mathematics only = 32
iii. Those who passed English $= 45 + 11 + 32 = 88$

Lead and guide learners to solve two-set problems of similar nature.
Let learners be involved in the class discussions.

Put them in groups and let them present their solutions to the class. Guide learners to name and label the subset regions and the universal set properly. Let learners solve further questions on two-set problems.

Expand and Box Method of Multiplication *(Refer to pages 43 and 44 of Learner's Book 8)*

The box method of multiplication is another form of long multiplication. In this method we split the factors and then continue with the multiplication.

Expanded refers to writing out each digit into its corresponding value. For example writing the 3-digit number 456 in its expanded form would be 400, 50 and 6.

Lesson Presentation
Lead and guide learners to use the expand and box method of multiplication.

Diagnostic Assesement
Find the product of 325 and 15

Solution

300	20	5	
300×10 $= 3000$	20×10 $= 200$	5×10 $= 50$	10
300×5 $= 1500$	20×5 $= 100$	5×5 $= 25$	5

$$\begin{aligned} 325 \times 15 &= 3000 + 1500 + 200 + 100 + 50 + 25 \\ &= 4500 + 300 + 75 \\ &= 4875 \end{aligned}$$

Addition of more than 4-digit numbers using Expanded Method *(Refer to page 45 of leaerners book 8)*
Lead and guide learners to add more than 4-digit numbers using expanded method.

Diagnostic Assessment
Find the sum of 93896 and 24358

$93896 + 24358$

By expansion, we have

$$\begin{aligned} 93,896 &= 90,000 + 3,000 + 800 + 90 + 6 \\ 24,358 &= \underline{20,000 + 4,000 + 300 + 50 + 8} \\ & \quad 110,000 + 7,000 + 1100 + 140 + 14 \\ & \quad 110,000 + 7,000 + 1000 + 100 + 100 + 40 + 10 + 4 \\ & \quad 110,000 + 8,000 + 200 + 50 + 4 \\ & \quad 118,000 + 250 + 4 \\ & \quad = 118,254 \end{aligned}$$

Therefore, $93896 + 24358 = 118,254$

Subtraction of more than four -digit numbers using expanded form

Refer to page 45 of Learner's Book 8.

Lead and involve learners to work the examples provided.

Diagnostic Assessement

Find 89453 – 42312

Solution

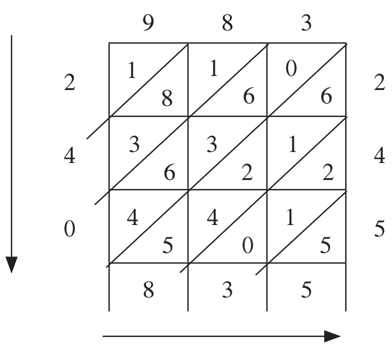
$$\begin{array}{rl} 89453 & = 80,000 + 9000 + 400 + 50 + 3 \\ -42312 & = \underline{40,000 + 2000 + 300 + 10 + 2} \\ & \quad 80.000 \quad 9,000 \quad 400 \quad 50 \quad 3 \\ & -40,000 + -2,000 + -300 + -10 + -2 \\ & \underline{\quad 40,000 \quad 7,000 \quad 100 \quad 40 \quad 1} \\ & \underline{\quad 47,141} \end{array}$$

Use the Expanded form and place value system to Add whole and decimal numbers. *(Refer to page 47 of LB)*

The Lattice Method of Multiplication *(Refer to pages 50-52 of Learner's Book 8)*

The lattice method of multiplication is used to multply whole numbers and integers. For example, find the product of 983 and 245.

983×245



Therefore, $983 \times 245 = 240,835$. Refer to pages 50-52 of learners book 8.

Guide learners to practice how to use the lattice method to do multiplication of whole numbers.

Box method of Division of multi digit numbers by 2 digit numbers. *(Refer to pages 53-54 of LB)*

Indices

Refer to pages 54-58 of Learner's Book 8
Indices are ways of representing repeated multiplication of numbers. For instance;
$2 \times 2 \times 2 \times 2 = 2^4 = 16$.

In an expression such as y^x, y is the base and x is called the index or power or exponent. Working with indices involves using rules which apply to any base. We therefore express these rules or propertise in terms of the base.

Rules of Indices

1. $y^m \times y^n = y^{m+n}$

   Eg. $3^5 \times 3^{10} = 3^{5+10} = 3^{15}$
   $5^{-1} \times 5^{10} = 5^{-1+10} = 5^9$

2. $y^m \div y^n = y^{m-n}$
   or $\frac{y^m}{y^n} = y^{m-n}$

   Eg. $3^5 \div 3^2 = 3^{5-2} = 3^3$
   $\frac{19^4}{19^{-2}} = 19^{4-(-2)} = 19^6$

3. $(y^m)^n = y^{mn}$
   Eg. $(2^7)^2 = 2^{14}$
   $(3^2)^2 \times 3^5 = 3^{4+5} = 3^9$

4. $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

   Eg. $\left(\frac{125}{8}\right)^{-\frac{1}{3}} = \left(\frac{8}{125}\right)^{\frac{1}{3}}$
   $= \frac{2^{3\times\frac{1}{3}}}{5^{3\times\frac{1}{3}}} = \frac{2}{5}$

Lead and guide learners to use the rules in solving the given examples. $y^{-n} = \frac{1}{y^n}$

Eg. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Properties of Indices

1. $y^0 = 1$
2. $\sqrt{y} = y^{\frac{1}{2}}$

Exponential Equations *(Refer to pages 57 and 58 of Learner's Book 8)*

$$y^m = y^n$$
$$m = n$$

NB. If the buses the same on either side, equate the exponents and solve. If the buses are not the same, we are logarithm to solve.

Eg. if $2^{x-1} = 8^{x+3}$, find the value of x.

$$2^{x=1} = 2^{3(x+3)}$$
$$x - 1 = 3(x + 3)$$
$$x - 1 = 3x + 9$$
$$x - 3x = 9 + 1$$
$$-2x = 10$$
$$x = -5$$

Involve learners to solve the examples given in the Learner's Book 8.

Sub-Strand/Unit 2: Number Operations

Refer to pages 62-68 of the Learner's Book 8

Content Standard
B8.1.2.1: Apply mental mathematics strategies and number properties used to solve problems.

B8.1.2.2: Apply the understanding of the addition, subtraction, multiplication and division of (i) Whole numbers within 10,000, and (ii) decimals up to 1/1000, to solve problems and round answers to given decimal places.

B8.1.2.3: Demonstrate understanding and the use of the laws of indices in solving problems (including real-life problems) involving powers of natural numbers.

Learning expectations:
After studying this sub-strand/unit, the learner will be able to:
- multiply and divide by power of 10 including decimals and benchmark fractions
- apply mental mathematics strategies and number properties to do calculations
- apply mental mathematics strategies to solve word problems

Keywords: *Refer to Learner's Book, page 62*
Guide learners to use the their dictionaries to find the contextual meaning of the keywords.
- *Benchmark fraction, product, halving, doubling*

Teaching and Learning Resources
A multiplication Chart for 12×12

Core Competencies
- Critical Thinking and Problem Solving
- Communication and Collaboration
- Creativity and Innovation

Multiplication chart for 12×12
Let learners read through the multiplication chart at Page 62 of Learner's Book 8. Lead and guide learners to use this multiplication chart to multiply and divide by power of 10.
For instance, how to multiply 6×4.

Guide learners to locate 6 on the first column and 4 along the first row. Now, let learners trace from along the row and also trace from 4 along the column. The answer will be at the intersecting cell, which is 24. Hence, $6 \times 4 = 24$

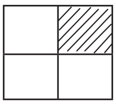
Benchmark fractions

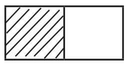
What are benchmark fractions?

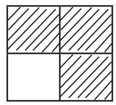
Benchmark fractions are the common fractions that can be measured or judged against when measuring, comparing or ordering other fractions. They are fractions that are easy to picture mentally. Examples are $\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$;

One-quarter, One-Half, Three-quarter

Revise learners previous knowledge on fractions briefly by drawing and shading portions a square represent these benchmark fractions,

1.  $\frac{1}{4}$ is shaded

2.  $\frac{1}{2}$ is shaded

3.  $\frac{3}{4}$ is shaded

Changing Fractions to Decimals *(Refer to page 64-65 of Learner's Book 8)*

Lead and guide learners to use the long division strategy to change fractions to decimals. For example; $\frac{1}{4}$ to decimal.

$$
\begin{array}{r}
0.25 \\
4\,\overline{)\;10} \\
\underline{-8} \\
20 \\
\underline{20} \\
--
\end{array}
$$

Therefore, $\frac{1}{4} = 0.25$

Changing Decimals to Fractions *(Refer to page 65 of Learner's Book 8)*

To change a decimal to a fraction, consider the place value of each digit.

Example:

1. Change 0.13 to a fraction

Solution

$$0.13 = 0 + \frac{1}{10} + \frac{3}{100} = \frac{10+3}{100} = \frac{13}{100}$$

2. Change 1.12 to a fraction

Solution

$$1.12 = 1 + \frac{1}{10} + \frac{2}{100} = 1 + \frac{10+2}{100} = 1 + \frac{12}{100}$$
$$= \frac{13}{25} = \frac{28}{25}$$

3. Convert 0.015 to a fraction

Solution

$$0.015 = 0 + \frac{0}{10} + \frac{1}{100} + \frac{5}{1000}$$
$$= \frac{1}{100} + \frac{5}{1000} = \frac{10+5}{1000}$$
$$= \frac{15}{1000} = \frac{3}{200}$$
$$\therefore 0.015 = \frac{3}{200}$$

Convert Fractions to percentages *(Refer to page 66 of Learner's Book 8)*
To change a fraction to percentage, we multiply the fraction by 100%.

Example 1 : Change $\frac{1}{4}$ to a percentage

Solution

$$\frac{1}{4} \times 100 = 25\%$$

Let learners study the table at page 65 of Learner's Book 8. Let learners solve the questions at exercise 2 of page 45 and discuss their solutions.

Convert from percentage to Fraction. *(Refer to page 67 of Learner's Book 8)*
Lead and guide learners to solve the questions at example 1
Put learners in groups of five to solve the questions at exercise 3.

For instance, $25\% = \frac{25}{100} = \frac{1}{4}$

Multiplication of a Decimal Number by Multiples of 10
(Refer to page 68 of Learner's Book 8) .
Multiplying a decimal by 10 moves the decimal point one place to the right. For each zero in

the power of 10, move the decimal point one place to the right.

Thus; $0.251 \times 10 = 2.51$

$0.00256 \times 1000 = 2.56$

Let learners solve the questions at exercise 4 page 68 of Learner's Book 8 and present their solutions.

Sub-Strand/Unit 3: Fractions, Decimals and Percentages.

Refer to pages 69-92 of Learner's Book 8.

Content Standard
B8.1.3.1: Apply the understanding of operation on fractions to solve problems involving fractions of given quantities and round the results to given decimals and significant places.

Learning expectations:
After studying this sub-strand/unit, the learner will be able to:
- solve problems involving basic operations on fractions
- add and/or subtract, multiply and/or divide given fractions by using the principle of order of operations (i.e. BODMAS or PEMDAS)

Keywords: *Refer to Learner's Book, page 69*
- *Fractions, BODMAS, PEMDAS, operation, parenthesis*

Core Competencies
- Critical Thinking and Problem Solving

Fractions
A fraction is a small part, amount or proportion of something. It is a numerical quantity that is not a whole number. Fraction can also represent parts of a set or collection. *Refer to page 74 of Learner's Book 8.* Let learners draw and shade fractions of shapes. Let learners discuss their shaded fractions with their colleagues.

Equivalent Fractions
Two or more fractions are equivalent if the value proportion or quantity they represent is the same. Equivalent fractions can have different numerators and denominators. Lead and guide learners to study the equivalent fractions given at examples 1-3 of Learner's Book 8.

Diagnostic Assessment
Let learners state three equivalent fractions of the following fractions;

1. $\frac{2}{3}$
2. $\frac{1}{5}$
3. $\frac{4}{7}$
4. $\frac{2}{5}$
5. $\frac{1}{6}$

Solutions

1. $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}$
2. $\frac{1}{5}=\frac{2}{10}=\frac{5}{25}=\frac{3}{15}$
3. $\frac{4}{7}=\frac{8}{14}=\frac{12}{21}=\frac{16}{28}$
4. $\frac{2}{5}=\frac{4}{10}=\frac{10}{25}=\frac{6}{15}$
5. $\frac{1}{6}=\frac{2}{12}=\frac{6}{36}=\frac{3}{18}$

Expressing fractions in simplest form *(Refer to page 71 of Learner's Book 8)*
To express a fractions in simplest form, let learners use a common factor to divide both numerator and denominator.

1. For example, express $\frac{12}{36}$ in simplest form. It can be seen that 12 is a common factor of 12 itself and 36. Hence 12 can be used to divide both 12 and 36. That is, $\frac{12}{36}=\frac{1}{3}$

2. $\frac{8}{48}=\frac{1}{6}$

Express improper fraction as a mixed number *(Refer to page 72-73 of Learner's Book 8)*
Explain what is improper fraction to learners. An improper fraction is a fraction whose numerator is greater than or equal to its denominator. Examples are; $\frac{12}{9},\frac{22}{18},\frac{15}{4},etc$

Let learners give further examples of an improper fraction.

Mixed Number
A mixed number is a fraction represented with its quotient and remainder or it is formed by combining a whole number and a fraction.

Examples are; $3\frac{1}{2},4\frac{3}{5},7\frac{1}{3}$ and so on

Let learners give other examples of mixed numbers.

Lesson presentation:
Lead and guide learners to express improper fractions as mixed number.

Guide learners through the following steps:

1. Divide the numerator by the denominator
2. Find the remainder
3. Arrange the numbers in the following way; quotient, followed by fraction of $\frac{Remainder}{Divisor}$

1. Express $\frac{7}{2}$ as a mixed fraction

Solution

2	7	R
	3	1

$$\frac{7}{2} = 3\frac{1}{2}$$

2. Express $\frac{12}{5}$ as a mixed number

Solution

5	12	R
	2	2

$$\frac{12}{5} = 2\frac{2}{5}$$

3. Express $\frac{23}{5}$ as a mixed number

Solution

5	23	R
	4	3

$$\frac{23}{5} = 4\frac{3}{5}$$

Involve learners to work through Examples 1 and 2.

Express mixed number as improper fraction *(Refer to page 73-74 of Learner's Book 8)*
Lead and guide learners through the procedure.

1. Multiply the whole number by the denominator
2. Add the product to the numerator
3. Write the new numerator over the original denominator

Example

1. Write $3\frac{1}{5}$ as an improper fraction

Solution

$$3\frac{1}{5} = \frac{(3 \times 5) + 1}{5} = \frac{16}{5}$$

2. Write as an improper fraction

Solution

$$5\frac{2}{3} = \frac{(5 \times 3) + 2}{3} = \frac{(15 + 2)}{3} = \frac{17}{3}$$

Put learners in groups of five to solve the questions at exercise 1.
Refer to pages 73 – 74 of Learner's Book 8.

Basic Operations on Fractions *(Refer to pages 75 – 76 of Learner's Book 8)*

1. Adding fractions with the same denominator

To add fractions having the same denominator ensure that;

i. The denominators of the fractions are the same
ii. Add the numerators and write the answer out of the denominator.
iii. Simplify the fractions (if possible)

Example; Find the sum of the following:

1. $\frac{1}{3}+\frac{2}{3}=\frac{3}{3}=1$
2. $\frac{5}{13}+\frac{2}{13}=\frac{7}{13}$
3. $\frac{9}{15}+\frac{1}{15}=\frac{10}{15}=\frac{2}{3}$
4. $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$
5. $\frac{23}{50}+\frac{19}{50}=\frac{42}{50}$
6. $\frac{20}{3}+\frac{5}{3}=\frac{25}{3}=8\frac{1}{3}$

Adding fractions with different denominators *(Refer to page 77 of Learner's Book 8)*
Guide and lead learners to use the least common multiple (L.C.M) strategy. For example find the sum of $\frac{3}{8}$ and $\frac{1}{5}$

Solution

$$\frac{3}{8}+\frac{1}{5}$$

$$\frac{15+8}{40}=\frac{23}{40}$$

Alternatively, we can use the concept of equivalent fractions whereby we can have same denominator.

$$\frac{(3\times 5)}{(8\times 5)}+\frac{(1\times 8)}{(5\times 8)}=\frac{15}{40}+\frac{8}{40}=\frac{23}{40}$$

Subtraction of like fractions
Refer to page 78 of Learner's Book 8

Lead and guide learners through the examples of *pages 77 – 78* of Learner's Book 8
Like fractions are fractions that have the same denominator.

Example;

1. Simplify $5\frac{1}{6} - 3\frac{2}{6}$

   Solution:
   Note that we can convert the mixed numbers to improper fractions and use the L.C.M to solve. Thus, $5\frac{1}{6} - 3\frac{2}{6}$

$$\frac{31}{6} - \frac{20}{6} = \frac{11}{6} = 1\frac{5}{6}$$

Alternative method

$$5\frac{1}{6} - 3\frac{2}{6}$$

$$(5-3) + \left(\frac{1}{6} - \frac{2}{6}\right)$$

$$2 + \left(\frac{-1}{6}\right)$$

$$\frac{2}{1} - \frac{1}{6}$$

$$\frac{12-1}{6} = \frac{11}{6} = 1\frac{5}{6}$$

2. Simplify $\frac{15}{19} - \frac{12}{19}$

   Solution

$$\frac{15}{19} - \frac{12}{19} = \frac{3}{19}$$

3. Simplify $\frac{23}{45} - \frac{19}{45}$

   Solution

$$\frac{23}{45} - \frac{19}{45} = \frac{4}{45}$$

Subtraction of fractions with different Denominators
Refer to page 80 of Learner's Book 8

Example

1. Simplify $\frac{5}{6} - \frac{2}{3}$

Solution

$$\frac{5}{6} - \frac{2}{3}$$
$$\frac{5-4}{6} = \frac{1}{6}$$

2. Simplify $\frac{3}{7} - \frac{2}{5}$

Solution

$$\frac{3}{7} - \frac{2}{5}$$
$$\frac{15-14}{35} = \frac{1}{35}$$

3. Simplify $\frac{12}{23} - \frac{2}{10}$

Solution

$$\frac{12}{23} - \frac{2}{10}$$
$$\frac{120-46}{230} = \frac{74}{230} = \frac{37}{115}$$

Give learners further examples to practice.

Multiplication of fractions *(Refer to page 81 of Learner's Book 8)*

1. Simplify $\frac{4}{7} \times \frac{21}{28}$

Solution

$$\frac{4}{7} \times \frac{21}{28} = \frac{3}{7}$$

When multiplying fractions, we can check to see whether the numerator and the denominator can be reduced to simpler numbers after which we can now multiply both numerators and denominators.

2. Simplify $3\frac{2}{3} \times 5\frac{3}{8}$

Solution

$$3\frac{2}{3} \times 5\frac{3}{8} = \frac{11}{3} \times \frac{43}{8} = \frac{473}{24} = 19\frac{17}{24}$$

3. Simplify $1\frac{2}{5} \times 2\frac{3}{7}$

Solution

$$1\frac{2}{5} \times 2\frac{3}{7} = \frac{7}{5} \times \frac{17}{7} = \frac{17}{5} = 3\frac{2}{5}$$

Application *(Refer to page 82 of Learner's Book 8)*

Example

Ama drank $\frac{8}{9}$ of a crate if malt this week. Dravid drank 10 times more malt than Ama. How many crates of malt did David drink? Write your answer as a fraction or as a whole numbers or a mixed number.

Solution

Ama drank $\frac{8}{9}$ of malt

David drank 10 times more than Ama. This implies that David will drink $10 \times \frac{8}{9}$ of malt $= \frac{80}{9} = 8\frac{8}{9}$

Lead and guide learners to solve the examples provided.

Division of Fractions *(Refer to page 83-84 of Learner's Book 8)*

Explain how to find the reciprocal of a fractions to leaners. The reciprocal of a fraction is done by interchanging the numerator and the denominator. For instance, the reciprocal of; $\frac{2}{5}\,is\,\frac{5}{2}, \frac{3}{13}\,is\,\frac{13}{3}$ and *so on*

1. Simplify $\frac{1}{7} \div \frac{3}{5}$

Solution

$$\frac{1}{7} \div \frac{3}{5}$$

$$\frac{1}{7} \times \frac{5}{3} = \frac{5}{21}$$

2. Simplify $3\frac{2}{3} \div 4\frac{1}{2}$

Solution

$$3\frac{2}{3} \div 4\frac{1}{2} = \frac{11}{3} \div \frac{9}{2}$$

$$\frac{11}{3} \times \frac{2}{9} = \frac{22}{27}$$

Diagnostic Assessment
Refer to page 82 of Learner's Book 8 and let learners solve the questions.
Order of operations

Order of Operations *(Refer to page 84-85 of Learner's Book 8)*
Involve learners to discuss the acronyms BODMAS and PEDMAS

BODMAS

$B \longrightarrow Bracket\,(\;\;)$

$O \longrightarrow Is\ order\ which\ refers\ to\ the\ numbers\ which\ involve\ powers, squareroots, (of)$

$D \longrightarrow Division(\div)$

$M \longrightarrow Multiplication(\times)$

$A \longrightarrow Addition(+)$

$S \longrightarrow Subtraction(-)$

PEDMAS

$P \longrightarrow Parenthesis(\;\;)$

$E \longrightarrow Exponents$

$D\ and\ M \longrightarrow Multiply\ and\ Divide(from\ left\ to\ right)$

$A\ and\ S\ Add\ or\ Subtract\ (from\ left\ to\ right)$

Lead and guide learners to solve examples 1,2 and 3. Refer to page 83 of Learner's Book 8

Diagnostic Assessment

Simplify $5\frac{2}{3} \div (23\frac{2}{5}\ of\ 3\frac{1}{4} - 22)$

Solution

$\frac{17}{3} \div (\frac{117}{5} of\ \frac{13}{4} - 22)$

$\frac{17}{3} \div (\frac{117}{5} \times \frac{13}{4} - 22)$

$\frac{17}{3} \div (15\frac{21}{20} - \frac{22}{1})$

$\frac{17}{3} \div \left(\frac{1521 - 440}{20}\right)$

$\frac{17}{3} \div \left(\frac{1081}{20}\right)$

$\frac{17}{3} \times \frac{20}{1081} = \frac{340}{3243}$

Solve Word problems involving Fractions *(Refer to page 90-91 of Learner's Book 8)*
Lead and guide learners to solve word problems involving fractions. Also, put learners in groups of five to solve and present word problems involving fractions.

Example: Martha spent $\frac{4}{9}$ of her allowance on food and shopping. What fraction of her allowance was left?

Solution

Note that, Fraction is an equal part of a whole. Therefore, the total fraction should be 1. Now, if she spent $\frac{4}{9}$ from the 1 on food and shopping, then she will be left with;

$$1 - \frac{4}{9} = \frac{5}{9}$$

$\therefore$ Martha had $\frac{5}{9}$ of her allowance left.

Lead and guide learners to solve the example under " Application of percentage of quantity"

Let learners solve the solve the exercise questions, discuss and present them.

Sub-Strand/Unit 4: Number Ratio and Proportion

Refer to page 93-116 of Learner's Book 8.

Content Standard
B8.1.4.1: Demonstrate an understanding of ratio, rate and proportions and use it these to solve real-world mathematical problems.

Let learners read and take note of the measurement units.

Learning Expectations:
After studying this sub-strand/unit, the learner will be able to:
- use ratio reasoning to convert measurement units, manipulate and transform units appropriately when multiplying or dividing quantities.
- solve unit rate problems including those involving unit pricing and constant speed and speed translation.
- apply the knowledge of speed to draw and interpret tracked graphs or distance time graphs.
- recognise and represent proportional relationships between quantities by deciding whether two quantities are in proportional relationship.
- identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationship.

Keywords: Refer to Learner's Book, page 93.
Guide learners to use the their dictionaries to find the contextual meaning of the keywords.
- *Ratio, proportional speed, rate pricing.*

Core Competencies
- Critical Thinking and Problem Solving
- Creativity and Innovation
- Personal Development and Leadership

Ratio *(Refer to page 93-94 of Learner's Book 8)*

Ratio is defined as the comparison of two quantities or numbers by division. For a ratio, the two quantities must be in the same unit.
The ratio of x *to* y is expressed as $x{:}y$. Also $\frac{x}{y}, y \neq 0, x{:}y = \frac{x}{y}$

When solving problems involving ratio, the order in which the quantities appear or are stated in the problem must be noted and followed accordingly.

Lead, guide and involve learners to solve the questions at example 1

Diagnostic Assessment

1. Change 20m to cm

Solution

If $100cm = 1m$, then $x = 20m$

We cross multiply and make x the subject

$$x = 20 \times 100$$
$$x = 2000$$

Hence $20m = 2000cm$

Applications

Example: Agbo walks 4 km to school everyday. He uses 60 minutes. Rukiya uses 45 minutes to cover 4200 m. Which of the two, walks faster?

Solution

Agbo's distance = 4 km, Time = 60 minutes
Rukiya's distance = 4200 m, Time 45 minutes

Ensure the distance covered is in the same unit of measurement. Preferably, convert the bigger unit to the smaller unit. Thus, change from Km to m.

$$1000m = 1km$$
$$x = 4km$$
$$x = 4 \times 1000 = 4000m$$

Therefore, Agbo's walking distance $= 4000m$. By comparing distance covered against time taken.

$Agbo: 4000m \longrightarrow 60$ minutes
$Rukiya: 4200m \longrightarrow 45$ minutes

It is obvious that Rukiya covered much more distance in less time. Hence Rukiya walks faster than Agbo.

Now we can also calculate their speed.

$$\text{Speed} = \frac{\text{Distance Covered}}{\text{Time Taken}}$$

$$60 \text{ minutes} = 3600 \text{ seconds}$$

Agbo's speed $= \frac{4000m}{3600s} = 1.11m/s$

Rukiya's speed $= \frac{4200m}{45 \times 60s} = \frac{14}{9}m/s$

$= 1.5556m/s$

This means 1.56 m/s is greater than 1.11 m/s. Therefore, Rukiya walks faster than Agbo.

Put learners in groups of five and let them solve questions at exercises A and B and let them present their solutions to the class.

Application of unit rate *(Refer to page 98-100 of Learner's Book 8)*
Discuss what is unit rate with the learners. Unit rate is a rate with 1 in the denominator.
If you have a rate such as price per a certain number of such items and quantity and the denominator is not 1, you can calculate unit rate or price per unit by completing the division operations. Numerator divided by denominator.

Example:
24 pencils cost $GH¢1.20$, how much will 6 pencils cost?

Price per pencil $= \frac{GH¢1.20 \div 24}{24\ pencils \div 24}$

$\frac{GH¢0.05}{Pencils} = 5$ Ghana Pesewas per pencil

This implies that 6 pencils will cost;

$$6 \times 0.05 = GH¢0.30 = 30GP$$

Example 2
Malik travelled 300 kilometers in 5 hours. Find the unit rate in kilometers per hour.

Solution:

Unit rate $= \frac{300Kilometers}{5hours}(km/h)$

$= \frac{60kilometers}{1hour} = 60km/h$

Guide and lead learners to solve other examples.

Travel graphs or Distance Time graphs *(Refer to pages 100 – 105 of Learner's Book 8)*
Lead and guide learners to study and discuss the examples at these pages. Let learners do the exercise in groups and present their solutions to the class.

Proportion *(Refer to pages 106-109 of Learner's Book 8)*

Proportion is an equation that defines that two given ratios are equivalent to each other. It states the equality of the two fractions or the ratios.

Again, a proportion is an equation in which two ratios are set equal to each other. For instance, if there is 1 boy and 3 girls you can write the ratio as 1:3.

This means, for every 1 boy, there are 3 girls. Also, we can have, of boys and of girls.

Two ratios (or fractions) under proportions can be equal. For example ,$\frac{1}{3} = \frac{2}{6}$ or $1:3 = 2:6$, hence the ratios are the same.

This means that, they are in proportion.

Example: A rope's length and weight are in proportion. When 20m of rope weight 1 kg, then;

i. 40m of that rope weights 2 kg
ii. 200m of that rope weighs 10 kg

.

Application of proportions
Refer to pages 109–114 of Learner's Book 8.
Lead and guide learners to solve the examples provided at the pages stated.

What is constant of proportionality?
When two variables or quantities are directly or indirectly proportional to each other then their relationship can be described as $y \propto x, y = kx$ or $y = \frac{k}{x}$ where K determines how the two variables are related to one another. This K is known as the constant of proportionality.

Constant of proportionality is the constant value of the ratio between two proportional quantities.

Two varying quantities are said to be in a relation of proportionality when either their ratio or their product yields a constant.
The value of the constant of proportionality depends on the type of proportion between the two given quantities. Direct variation and inverse variation.

Diagnostic Assessment

Find the constant of proportionality from the table of values.

Marks scored (m)	1	2	3	4	5
No. of Students (s)	12	24	36	48	60

Solution

Constant of proportionality

$$K = \frac{No.\ of\ student}{Marks} = \frac{24}{2} = 12$$

$$K = \frac{36}{3} = 12$$

$$K = \frac{48}{4} = 12$$

This implies that the number of students in relation to the marks obtained varies by a constant of K = 12. Thus, every mark obtained is proportional to 12 additional students.

Let learners solve the questions at Exercise 6 page 105 of Learner's Book 8.

STRAND/CHAPTER 2: ALGEBRA

Sub-Strand/Unit 1: Patterns and Relations

Refer to page 117-139 of the Learner's Book 8

Content Standards
B8.2.1.1: Demonstrate the ability to draw a table of values for a linear relation, graph the relation in a number plane, determine the gradient of the line and use it to write an equation of a line of the form $y = mx + c$.

Learning Expectations:
After studying this sub-strand/unit, the learner will be able to:
- calculate the gradient/slope of a line and use it to write the equation of a line in the form $y = mx + c$
- determine the slope-intercept form of the equation of a straight line
- use graph of linear relation to determine subsequent missing elements in the ordered pairs of relation
- use graphs of linear relations to solve real life problems.

Keywords: *Refer to Learner's Book page 117*
Help learners to use the their dictionaries to find the contextual meaning of the keywords.
- *gradient, linear equation, straight line*

Core Competencies
- Critical Thinking and Problem Solving
- Personal Development and Leadership
- Creativity and Innovation

Discuss the keywords with the learners. Gradient or slope of a line shows how steep it is. To calculate the gradient/slope of a line, divide the change in height by the change in horizontal.

Again, the gradient is calculated by dividing the difference in the $y - coordinates$ by the difference in the $x - coordinates$

The gradient-intercept form of the equation of a line is given by $y = mx + c$ where m is the gradient and c is the $y - intercept$

Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$. This may be referred to as change in y divided by change in x. The gradient is a measure of slope. The greater the gradient the steeper the slope. When the gradient of two lines are the same, then the lines are parallel.

When the gradients of two lines have a product of -1, then the lines are perpendicular.

Straight Line

A line is a straight one-dimensional figure having no thickness and extending infinitely in both directions. A line is sometimes called a straight line or more archaically a right line (Casey 1893) to emphasise that is has no "wiggles" anywhere along its length. In geometry, a line is an infinitely long object with no width, depth, or curvature. Thus, lines are one-dimensional objects, though they may exist embedded in two, three or higher dimension spaces.

Linear Equation:

A linear equation is an algebraic equation of the form $y = mx + c$, involving only a constant and a first-order (linear) term, where m is the gradient or slope and c is the $y - intercept$. Sometimes $y = mx + c$ is called a linear equation of two variables, where are the two variables. The standard form for a linear equation in two variables is $ax + by + c = 0$

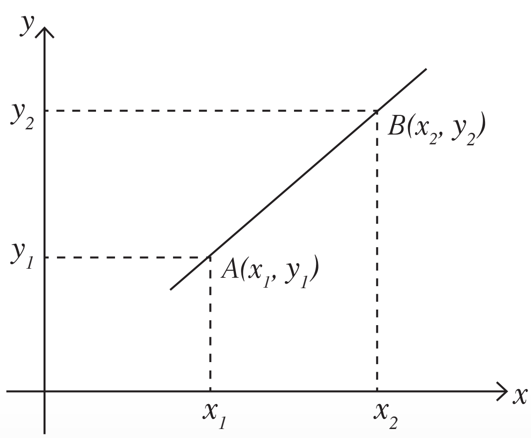
For example $2x + 3y + 4 = 0$ is a linear equation in standard form. When a linear equation is given in this form, it is easy to find the x and y intercepts.

Gradient of a Non-Vertical Straight Line *(Refer to page 118 of Learner's Book 8)*

Let learners mention and discuss other examples of non-vertical straight lines.

i. Roofing of a building
ii. Staircase to a storey building
iii. Supporting wires for a high tension pole
iv. A ladder placed to lean on a wall
v. The view from the foot of a mountain to the top of the mountain
vi. The windscreen of a vehicle
vii. The leaves on a palm tree

All these examples and many others depict slope or gradient which tell how steep a line is.



To find the gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 - x_1 \neq 0$

How to determine the gradient when two coordinates are given

Refer to page 119-120 of Learner's Book 8

To find the gradient of a line, we need any two points on the line. If the coordinates of two points say, A(3,4) and B(6,7) then the gradients is given by;

$$m = \frac{7-4}{6-3} = \frac{3}{3} = 1 \text{ or } m = \frac{4-7}{3-6} = \frac{-3}{-3} = 1$$

Diagnostic Assessment

1. If a line PQ passes through the point $(0, -5)$ and $(-3, 10)$, find the gradients of the line.

   Solution

   Gradient/slope $m = \frac{10-(-5)}{-3-0} = \frac{15}{-3} = -5$ or $m = \frac{-5-10}{0-(-3)} = \frac{-15}{3} = -5$

2. Given the gradient of a line $\overline{AB} = \frac{1}{2}$ find the value of x if $A(x,4)$ *and* $B(6,3)$

   Solution

   Gradient $m = \frac{4-3}{x-6}$

   $m = \frac{1}{2} = \frac{1}{x-6}$ (*Cross multiply*)

   $x - 6 = 2$

   $x = 2 + 6$

   $x = 8$

3. The coordinates $P(3,y)$ and $Q(9,12)$ are two points on a line where slope is 5. Find the value of y.

   Solution

   $m = \frac{12-y}{9-3}$

   $\frac{5}{1} = \frac{12-y}{6}$ (*Cross multiply*)

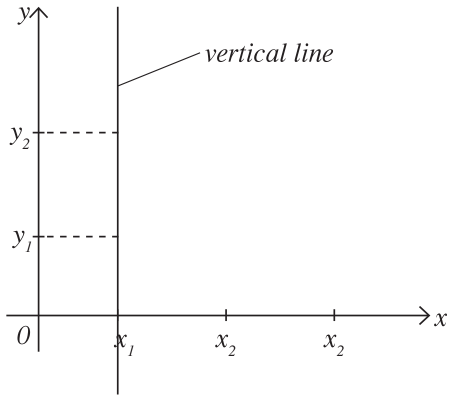
   $5 \times 6 = 12 - y$

   $30 = 12 - y$

   $y = 12 - 30$

   $y = -18$

Gradient of a Vertical Line *(Refer to page 121 of Learner's Book 8)*
The gradient of a vertical line is undefined. This means that a vertical line has no slope/ gradient.

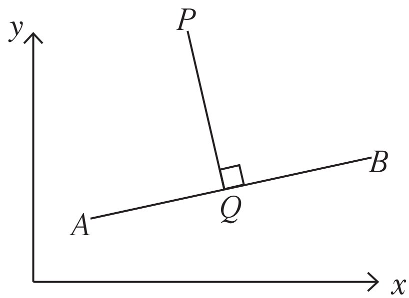


From the above, we have change in y but no change in x. This implies that the vertical line has no gradient/slope

Lead and guide learners to solve examples at page 112 of Learner's Book 8

Gradient of parallel lines *(Refer to page 123 of Learner's Book 8)*
Parallel lines are in a plane that are always the same distance apart. Parallel lines never intersect. They can be both horizontal and vertical. Example of parallel line in real life is a pedestrian crossing, dual carriage highways, etc.
Parallel lines have the same gradient.

Gradient of perpendicular lines *(Refer to page 124 of LB)*
Perpendicular lines are lines that intersect at a right angle (i.e. $90°$). This is often shown by a right angle symbol in the corner where the two lines meet. We can see perpendicular lines everywhere, from the corners of buildings to branches in the trees. Also the corners of tables and books. The product of the gradients of perpendicular lines is -1.



The line $\overline{AB}$ is perpendicular to the line $\overline{PQ}$. That is $\overline{AB} \perp \overline{PQ}$

Diagnostic Assessment

1. Show that the line passing through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line passing through the points $(2,3)$ and $(1,6)$.

Solution:

For points $(-2,6)$ and $(4,8)$

$$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

For points $(2,3)$ and $(1,6)$

$$m_2 = \frac{6-3}{1-2} = \frac{3}{-1} = -3$$

If they are perpendicular then; $\frac{1}{3} \times \frac{-3}{1} = -1$

This implies that the lines passing through the points (-2,6) and (4,8) and (2,3) and (1,6) are perpendicular.

Refer to pages 114 – 116 of Learner's Book 8 and put learners in groups of five to solve the questions and discuss their solutions. Give learners some other questions on how to find gradient. Let them solve the questions and present their solutions.

Determine the Gradient from a Graph *(Refer to pages 126 – 127 of Learner's Book 8)*
Lead and guide learners to draw the graph of a line given two coordinates and let them determine the gradient.

The slope-intercept form of the equation of a straight line
Refer to page 128-129 of Learner's Book 8.
Guide and lead learners to state and use the gradient – intercept form of the equation of a straight line, $y = mx + c$ where m is the gradient, c is the $y-intercept$ x and y are variables of x *and* y. Give equations of a line and involve the learners to determine; gradient and $y-intercept$

Example: State the gradient and $y-intercept$ of;

i. $3y = x - 2$
ii. $y - 3x + 1 = 0$
iii. $2x - y - 10 = 0$

Solutions

i. $3y = x - 2$

Rearrange the given equation by making y the subject and compare it to the gradient-intercept form to enable you to determine m and c

$$3y = x - 2 (\textit{divide each term by } 3)$$

$$y = \frac{x}{3} - \frac{2}{3}$$

By comparing with $y = mx + c$

$$m = \frac{1}{3} \text{and } c = -\frac{2}{3}$$

ii. $y - 3x + 1 = 0$

$$y = 3x - 1$$

Gradient $m = 3$ and $y - intercept\ c = -1$

iii. $2x - y - 10 = 0$

$$y = -2x + 10$$

Gradient $m = -2$ and $y - intercept\quad c = 10$

Again, lead and guide learners to write the equation of a line in a gradient – intercept form when given the slope and $y - intercept$.

Example: Write down the equation of a line whose gradient is 2 and $y - intercept$ is -3.

Solution

Given the gradient, $m = 2$ and $y - intercept\quad c = -3$ we have; $y = 2x - 3$

Let learners practice by solving the examples and exercises provided at these pages. Let learners present and discuss their solutions.

Diagnostic Assessment

Write the equation of a line with the following gradients and intercepts;

1. Slope is $\frac{1}{2}$ and $y - intercept\ -2$
2. Gradient is $y - intercept\ -3$ and is $\frac{1}{3}$
3. Gradient is $-\frac{3}{5}$ and $y - intercept$ is 0
4. Slope is 10 and $y - intercept$ is 19
5. Slope is -1 and $y - intercept$ is 5

How to determine the point-gradient form of the equation of a line

We can write the equation of a line when the gradient and a point of a line are given. The point – gradient form of the equation of a line is given by $y - y_1 = m(x - x_1)$ where m is the gradient and (x_1, y_1) is the point.

Example: Find the equation of a line having a gradient 3 and passes through the point$(3,9)$

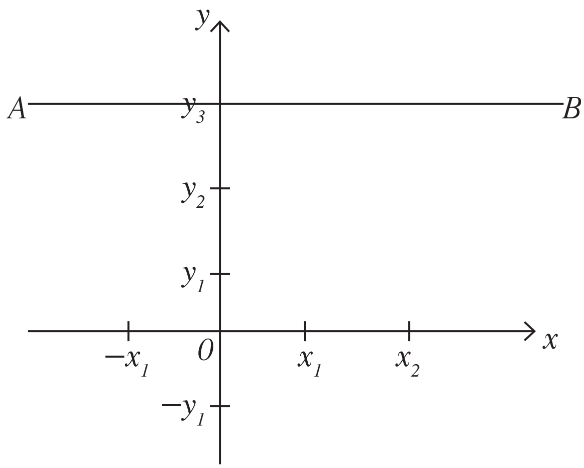
Solution:

Gradient $m = 3$
Point $(3,9)$

$$y - y_1 = m(x - x_1)$$
$$y - 9 = 3(x - 3)(\textit{Expand and Regroup})$$
$$y - 9 = 3x - 9$$
$$y - 3x = -9 + 9$$
$$y = 3x + 0$$
$$y = 3x$$

Lead and guide learners to solve the questions provided at the examples on pages 120 – 132 of Learner's Book 8.

Gradient of a horizontal line



The line $\overline{AB}$ is a horizontal line that passes through y_3 It can be seen that, there is no change in the $y - values$. That is $y_2 - y_1 = 0$

Hence, the gradient of a horizontal line is zero. Gradient of line $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1} = x_2 \neq x_1$

Gradient of $\overline{AB} = \frac{0}{x_2 - x_1} = 0$

Equation of a horizontal line *(Refer to page 122 of Learner's Book 8)*
To find the equation of a horizontal line passing through the point $(0, 4)$

Solution

The gradient of a horizontal line, $m = 0$

$$y - 4 = m(x - 0)$$
$$y - 4 = 0(x - 0)$$
$$y = 4$$

Involve learners to solve the questions at *pages 124 –129 of Learner's Book 8.*

Sub-Strand/Unit 2: Algebraic Expressions

Refer to pages 140 – 161 of Learner's book 8

Content Standards
B8.2.2.1: Solve problems involving algebraic expressions (including multiplication of binomial expressions) factorise given expressions and substitute values to evaluate algebraic expressions.

Learning expectations:
After studying this sub-strand/unit, the learner will be able to:
- use the distributive property to solve multiplication of binomial expressions
- add, Subtract Multiply and Divide algebraic expressions
- substitute values into algebraic expressions and evaluate them

Keywords: *Refer to Learner's Book 8, page 140.*
Lead learners to use the their dictionaries to find the contextual meaning of the keywords.
- *distributive, denominator, numerator, lowest common multiple, coefficient, operand*

Core Competencies
- Critical Thinking and Problem Solving
- Creativity and Innovation
- Personal Development and Leadership

Lesson Presentation
Explain what is distributive property to learners. According to the distributive property, multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the product together.

In other words, according to the distributive property, an expression of the form $a(b+c)$ can be solved as $ab+ac$.

Thus, $a(b+c)=ab+ac$

We can apply this property to subtraction

$$a(b-c)=ab-ac$$

This indicates that operand 'a' is shared between the other two operands.

Example; Simplify $4(3+7)$

Solution

$4(3+7)$

$12+28=40$

The distribution law of multiplication over basic arithemetic, such as addition and subtraction is known as the distributiive property.

Diagnostic Assessment

1. Expand and simplify: $3(x+5)+2(x-3)$

Solution

$3(x+5)+2(x-3)$

$3x+15+2x-6$

$3x+2x+15-6$ $(group\ like\ terms)$

$5x+9$

2. $(y-3)-2(y-4)$

Solution

$(y-3)-2(y-4)$

$y-3-2y+8$

$y-2y+8-3$

$-y+5=5-y$

3. $3(m-5)+6(m-2)$

Solution

$3(m-5)+6(m-2)$

$3m+6m-15-12$

$9m-27$

Multiplying and simplifying binomial expressions

Refer to page 143-146 of Learner's Book 8.

Binomial expressions are algebraic expressions that contain only two terms called binomial.

Examples are; $(x+y),(3x-y),(a^2+b)$ etc.

Multiply

i. $(a+b)(c+d)$

$$(a+b)(c+d)$$
$$a(c+d)+b(c+d)$$
$$ac+ad+bc+bd$$

ii. $(a-b)(c-d)$

$$(a-b)(c-d)$$
$$a(c-d)-b(c-d)$$
$$ac-ad-bc+bd$$

Lead and guide learners to solve the given examples.

1. Expand and simplify $(2x-3y)(x-2y)$

Solution

$$(2x-3y)(x-2y)$$
$$2x(x-2y)-3y(x-2y)$$
$$2x^2-4xy-3xy+6y^2$$
$$2x^2-7xy+6y^2$$

2. Multiply $(2-m)by(3+m)$

Solution

$$(2-m)(3+m)$$
$$6+2m-3m-m^2$$
$$6-m-m^2$$

Let learners solve the examples at *pages 143 – 146 of Learner's Book 8.* Let learners present and discuss their solutions.

Multiplication and Division of Algebraic expressions.
Refer to page 146 of Learner's Book 8

Lead and guide learners to solve the example 1.

1. Simplify $\frac{x}{2} \times \frac{14}{3}$

   Solution $\frac{x}{\not{2}} \times \frac{\not{14}^{7}}{3} = \frac{7x}{3}$

2. Simplify $\frac{y}{3} \times \frac{9}{2} \times \frac{1}{6}$

   Solution $\frac{y}{\not{3}} \times \frac{\not{9}^{3}}{2} \times \frac{1}{\not{6}_{2}} = \frac{y}{4}$

3. Simplify $\frac{x-1}{2x-4} \times \frac{x-2}{2(x-1)}$

   Solution

   $$\frac{x-1}{2x-4} \times \frac{x-2}{2(x-1)}$$

   $$\frac{\cancel{(x-1)}}{2\cancel{(x-2)}} \times \frac{\cancel{(x-2)}}{2\cancel{(x-1)}} = \frac{1}{4}$$

Involve learners to solve other examples

Addition and subtraction of Algebraic Fractions with different denominators
Refer to page 149-153 of Learner's Book 8

When algebraic fractions have different denominators, we follow the steps below to solve it.

i. Find the least common multiple (LCM) of the denominators.
ii. Divide the LCM by each denominator and multiply the quotient obtained by the corresponding numerator
iii. Carry out the operations of addition or subtraction in the numerator
iv. Simplify the result by reducing it to lowest terms.

Read and guide learners to solve some examples.

Example 1. Simplify $\frac{2}{b-1}-\frac{1}{b}$

Solution

$$\frac{2}{b-1}-\frac{1}{b}$$

$$\frac{2b-1(b-1)}{b(b-1)}$$

$$\frac{2b-b+1}{b(b-1)}=\frac{b+1}{b(b-1)}$$

Example 2. Simplify $\frac{3}{x+1}+\frac{4}{x-1}$

Solutiion

$$\frac{3}{x+1}+\frac{4}{x-1}$$

$$\frac{3(x-1)+4(x+1)}{(x+1)(x-1)}$$

$$\frac{3x-3+4x+4}{(x+1)(x-1)}$$

$$=\frac{7x+1}{x^2-1}$$

Example 3. Simplify $\frac{x-1}{3-x}-\frac{3}{x-1}$

Solution

$$\frac{x-1}{3-x}-\frac{3}{x-1}$$

$$\frac{(x-1)(x-1)-3(3-x)}{(3-x)(x-1)}$$

$$\frac{x^2-2x+1-9+3x}{(3-x)(x-1)}$$

$$=\frac{x^2+x-8}{(3-x)(x-1)}$$

Put learners in groups to solve the exercises and let them discuss their solutions.

Evaluation of Algebraic Expressions *(Refer to page 154-157 of Learner's Book 8)*
Lead and guide learners to substitute values given into an algebraic expression and evaluate it.

Give opportunity to learners to lead the class to substitute given values into an algebraic expression and evaluate it.

Example 1. Given that $x = 3, y = 2$ and $t = 4$

Evaluate $\frac{3x - yt}{y - 2t}$

$$\frac{3(3) - 2(4)}{2 - 2(4)}$$
$$= \frac{9 - 8}{2 - 8}$$
$$= \frac{1}{-6}$$

Example 2. Given that $\frac{x - y - t}{t - xy}$, evaluate the expression if $x = -1$, $y = 2$ and $t = 5$

Solution

$$\frac{x - y - t}{t - xy}$$
$$= \frac{(-1) - 2 - 5}{5 - (-1)(2)}$$
$$= \frac{-8}{5 + 2}$$
$$= \frac{-8}{7}$$

Refer to *page 145 – 147 of Learner's Book 8* and put learners in groups to practice solve questions and present their solutions.

Methods of Factorising Algebraic Expressions

Refer to page 158-159 of Learner's, Book 8.

Factorisation of an Algebraic expression is writing the given expression or equation as a product of its factors. These factors can be such that when multiplied together they will result in the original algebraic expression.

What is a factor?

A factor is an algebraic expression that is to be multiplied in an expression.

1. Factorize $2x - 4$

   Solution: $2x - 4 = 2(x - 2)$

2. Factorize $4x^2 - 16x$

   Solution: $4x(x - 4)$

3. Factorize $\frac{1}{2}x^2 - \frac{1}{4}x$

Solution

$$\frac{1}{2}x^2 - \frac{1}{4}x$$
$$= \frac{1}{2}x\left(x - \frac{1}{2}\right)$$

Regroping of terms method

Refer to page 159-160 of Learner's Book 8.

This method of grouping terms is the strategy of putting like or common terms together when solving an algebraic expression.

Let learners identify algebraic terms that have common or like variables. Also unlike or uncommon terms are solved as such.

Example 1

Factorise the expression $3ab + 4b + 3a + 4$

Solution

$$3ab + 3a + 4b + 4 (\textit{Rearranging the terms})$$
$$(3ab + 3a) + (4b + 4)$$
$$3a(b + 1) + 4(b + 1)$$
$$(b + 1)(3a + 4)$$

Example 2

Factorise $xy - 2x + 8y - 4xy$

Solution

$$xy - 2x + 8y - 4xy$$
$$xy - 4xy - 2x + 8y$$
$$-3xy - 2x + 8y$$
$$8y - 3xy - 2x$$
$$y(8 - 3x) - 2x$$

Involve learners to solve other examples.

Factorising using Identities *(Refer to page 160 – 161 of Learner's Book 8)*

By using certain common identities, we can factorise the given algebraic expressions.
Revise the concept of difference of two squares with learners.

Difference of Two squares

This concept presents us with an identity that helps to factorise certain algebraic expressions.

It is stated in the form; $x^2 - y^2 = (x + y)(x - y)$

$$x^2 - 4 = x^2 - 2^2 = (x + 2)(x - 2)$$
$$25 - y^2 = 5^2 - y^2 = (5 + y)(5 - y)$$
$$4x^2 - 100 = (2x)^2 - 10^2 = (2x + 10)(2x - 10)$$
$$64m^2 - 121 = (8x)^2 - 11^2 = (8x + 11)(8x - 11)$$
$$4y^2 - 1^2 = (2y)^2 - 1 = (2y + 1)(2y - 1)$$

Lead and guide learners to solve the questions at example 2, *page 151 of Learner's Book 8*. Let learners solve the exercise questions and present their solutions to the class.

Sub-Strand/Unit 3: Variables and Equations

Refer to pages 162 – 173 of Learner's Book 8.

Content Standards
B8.2.3.1: Demonstrate an understanding of linear inequalities of the form $x + a \geq b$ (where a and b are integers) by modelling problems as linear inequalities and solving the problems concretely, pictorially, and symbolically.

Learning expectations:
After studying this sub-strand/unit, the learner will be able to:
- translate word problems into linear inequalities in one variable and vice versa.
- solve simple linear inequalities
- determine solution sets of simple linear inequalities in the given domains

Keywords: *Refer to Learner's Book 8, page 162.*
Guide learners to use the their dictionaries to find the contextual meaning of the keywords.
- *Less than, greater than, inequality, numerical statement*

Applications of linear inequalities

Core Competencies
- Critical Thinking and Problem Solving
- Creativity and Innovation

Lesson Presentation:
Let learners identify the inequality symbols:

$$' < ' \longrightarrow \textit{Less than}$$
$$' > ' \longrightarrow \textit{Greater than}$$
$$' \leq ' \longrightarrow \textit{Less than or equal to}$$
$$' \geq ' \longrightarrow \textit{Greater than or equal to}$$
$$' \neq ' \longrightarrow \textit{Is not equal to}$$

Inequalities are defined as a mathematical expression in which two numerical expressions or algebraic expressions are compared using the inequality symbols $\geq, <, >$ or $\leq$

Examples: $5x + 2 < x + 8, 3 - x > \frac{3}{2} + 3x, x - 1 \leq 7, \frac{x}{2} < 4(2x - 1)$

The inequality symbols $>$ *and* $<$ denote the strict inequalities and the symbols $\geq$ *and* $\leq$ represent the slack inequalities. *Refer to page 163 of Learner's Book 8.*

Solving linear inequalities in one variable problems
Refer to page 163-168 of Learner's Book 8.
Note the following carefully.

1. Reverse the inequality sign whenever you divide or multiply both sides of the inequality by – 1. For instance; $3 - x < 5$

$$3 - x < 5$$
$$-x < 5 - 3$$
$$-x < 2$$
$$x > -2$$

(We reverse the inequality sign from $< to >$ because we divided or multiplied both sides by -1)
When solving problems involving inequalities one must note the following expressions and their mathematical interpretations.

1. *at least* $\Rightarrow \geq$
2. *not more than* $\Rightarrow \leq$
3. *at most* $\Rightarrow \leq$
4. *more than* $\Rightarrow >$
5. *Less than* $\Rightarrow <$

Domain of Linear Inequalities *(Refer to pages 168-173 of LB)*
The domain of a linear inequality is always all real numbers, regardless of the sign of inequality.

Domain is the set if all values, the independent quantity for which the function $f(x)$ exists or is defined.

Lesson Presentation
Lead learners to solve examples of how to find the domain of linear inequalities.

Example 1. Find the domain for the inequality $2x > 12$

Solution

$2x > 12$

Divide both sides by 2

$$\frac{2x}{2} > \frac{12}{2}$$
$$x > 6$$

Domain for the solution is $\{x{:}x > 6\}$

Example 2. Find the domain for the inequality $\frac{1}{3}x - 1 < 5$

Solution

$$\frac{1}{3}x - 1 < 5$$

Multiple through (term by term) by 3 to clear fraction

$$3 \times \frac{1}{3}x - 1 \times 3 < 3 \times 5$$
$$x - 3 < 15 (\textit{group like terms})$$
$$x < 15 + 3$$
$$x < 18$$

Domain is $\{x : x < 18\}$

Refer to page 168 -172 of Learner's Book 8

Diagnostic Assessment

If 5 times a number is increased by 9, the result is at least 24. Find the least possible number that satisfies these conditions.

Solution: Let x represent the unknown number.

5 times a number $\Rightarrow 5x$
Is increased by $9 \Rightarrow 5x + 9$
The result is at least $24 \Rightarrow \geq 24$

Thus, $5x + 9 \geq 24$

$$5x \geq 24 - 9$$
$$5x \geq 15$$
$$\frac{5x}{5} \geq \frac{15}{5}$$
$$x \geq 3$$

Domain $\{x : x \geq 3\}$

Let learners practice how to solve problems of this nature individually and in groups. Guide learners to solve the examples provided at *page 170 – 172 of Learner's Book 8.*

STRAND/CHAPTER 3: GEOMETRY & MEASUREMENT

Sub-Strand/Unit 1: Shapes and Space

Refer to Learner's Book 8, pages 174-205.

Content Standards
B8.3.1.1; Demonstrate understanding and use of the relationship between parallel lines and alternate and corresponding angles and use the sum of angles in a triangle to deduce the angle sum in any polygon.

B8.3.1.2: Demonstrate the ability to perform geometric constructions of the angles ($75°$, $105°, 60°, 135°$ *and* $150°$), and construct triangles and find the locus of points under given conditions.

Learning Expectations:
After studying this sub-strand/unit, the learner will be able to:
- measure a line with a ruler and indicate the unit in centimetre (cm)
- measure angles accurately with a protractor
- construct various angles with a pair of compasses
- bisect angles with a pair of compasses
- deduce the formula for finding the interior angle of a polygon
- divide an angle into two and also lines into two
- divide polygons into triangles

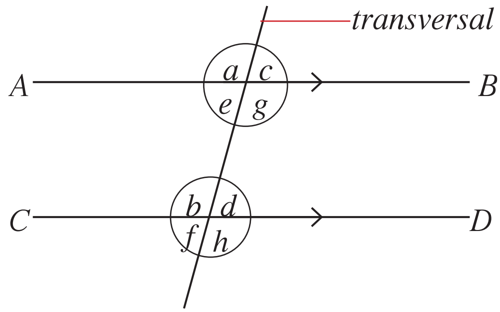
Keywords: *Refer to Learner's Book 8, page 174.*
Guide learners to use the their dictionaries to find the contextual meaning of the keywords.
- *polygons, pentagon, hexagon, formula, geometric, transversal construction, bisector, triangle, locus, circle and perpendicular*

Core Competencies
- Creativity and Innovation

What is a polygon?
In geometry, a polygon can be defined as a flat or plane, two-dimensional closed shape bounded with straight sides. It does not have curved sides. The sides of a polygon are called edges.

The point where two sides or edges meet are the vertices (or corners) of a polygon. Example of a polygon is a triangle, pentagon, heptagon, square, rectangle and so on.

A polygon is a closed figure made up of line segments (not curves) in a two-dimensional plane. Polygon is the combination of two words; that is; poly (meaning many) and gon (meaning sides). A minimum of three line segments are required to connect end-to-end to make a closed figure.



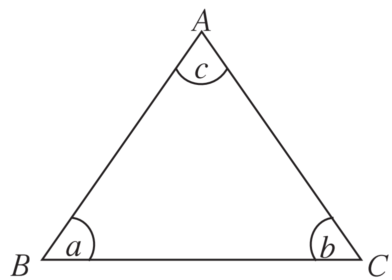
Line $\overleftrightarrow{AB}$ is parallel to line $\overleftrightarrow{CD}$

Lead and guide learner's to identify;

i. Corresponding angles
ii. Alternate angles
iii. Vertically opposite angles
iv. Alternate interior angles
v. Alternate exterior angles

Interior Angles in a Triangle and other Properties

Refer to page 184 of Learner's Book 8.

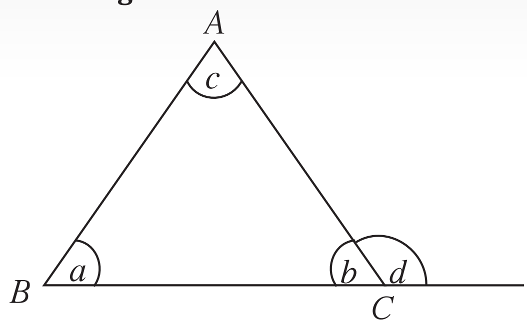


The interior angles are $\angle a, \angle b,$ *and* $\angle c$

Sum of interior angles is 180°

$$\angle a + \angle b + \angle c = 180^\circ$$

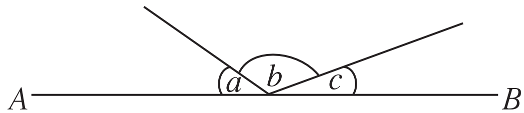
Exterior angle property of a triangle



$\angle d$ is an exterior angle

1. At a vertex the sum of one exterior and one interior angle at a vertex is equal to $180°$. That is $\angle d + \angle b = 180°$

2. One exterior angle is equal to the sum of two opposite interior angles. That is $\angle d = a + c$

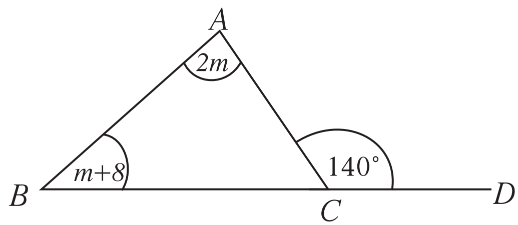


Sum of angles on a straight line is equal to $180°$. That is $\angle a + \angle b + \angle c = 180°$

Lead and guide learners to solve the example at page 185 of Learner's Book.

Diagnostic Assessment

1. Find the value of m in the diagram below



Solution:

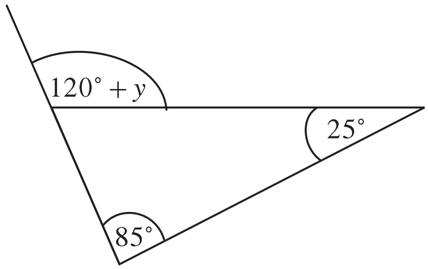
$$140° = m + 8 + 2m$$
$$140° = 3m + 8$$
$$140° - 8 = 3m$$

$132^\circ = 3m$

$\frac{132}{3} = m$

$44^\circ = m$

2. Find the value of y in the diagram below



Solution

$120^\circ + y = 85^\circ + 25^\circ$

$120^\circ + y = 110^\circ$

$y = 110^\circ - 120^\circ$

$y = -10^\circ$

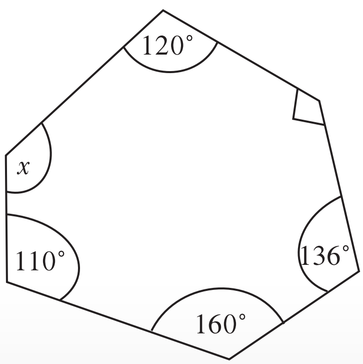
Polygon

Refer to page 186 of Learner's Book 8.
Lead and guide learners to find the sum of the internal angles of a polygon.

The sum of the internal angles of a polygon is given by $(n-2) \times 180^\circ$ where n is the number of sides of the polygon.

Diagnostic Assessment

1. Find the value of x in the diagram below



Solution

The diagram above is a hexagon, therefore the number of sides $n = 6$. The sum of interior angles is given by,

$$s = (n - 2) \times 180^\circ$$

$$x + 90^\circ + 120^\circ + 110^\circ + 160^\circ + 136^\circ = (6 - 2) \times 180^\circ$$
$$x + 616 = 4 \times 180^\circ$$
$$x = 720 - 616$$
$$x = 104^\circ$$

Construction of Angles

Refer to page 189 – 205 of Learner's Book 8.

Lead and guide learners to construct the following angles;

$$60^\circ, 30^\circ, 15^\circ, 90^\circ, 45^\circ, 120^\circ \textit{and } 75^\circ$$

Let learners use their pair of compasses and ruler to do these construction of angles. Let learners, do the constructions individually.

To bisect a line or an angle means that divide the line or angle into two equal parts. Guide learners to avoid double lines when doing their construction.

Specific angles constructed should be labelled.

Note that certain angles are a sum of two angles and learners must be made aware. For instance;

$$90^\circ = 45^\circ + 45^\circ (\textit{bisect } 90^\circ \textit{ to obtain } 45^\circ)$$
$$60^\circ = 30^\circ + 30^\circ (\textit{bisect } 60^\circ \textit{ to obtain } 30^\circ)$$
$$120^\circ = 60^\circ + 60^\circ$$
$$75^\circ = 60^\circ + 15^\circ$$
$$15^\circ = 30^\circ \div 2 (\textit{bisect } 30^\circ \textit{ to obtain } 15^\circ)$$

Sub-Strand/Unit 2: Measurement

Refer to page 206-235 of Learner's Book 8.

Content Standards
B.8.3.2.1: Apply the Pythagoras theorem, the primary trigonometric ratios and the formulas for determining the area of a circle to solve real problems.

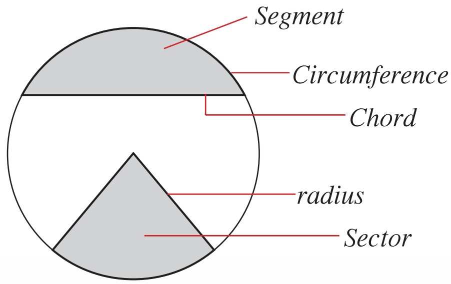
B8.3.2.2: Demonstrate understanding of addition and subtraction of vectors and their applications in solving basic problems

Learning expectations:
After studying this sub-strand/unit, the learner will be able to:
- divide a circle into sectors (minimum of 16)
- solve problems on area of a circle
- construct squares on three sides of a right-angled triangle in a square grid
- solve problems involving Pythagoras Theorem
- identify and recognise three primary trigonometric ratios
- add, Subtract and Find scalar multiplication of vectors in the component form

Keywords: *Refer to Learner's Book 8, page 206.*
Help learners to use the their dictionaries to find the contextual meaning of the keywords.
- *circumference, hypotenuse, opposite, adjacent, pythagoras, sine, cosine, tangent, SOH-CAH-TOA, angle of elevation, depression*

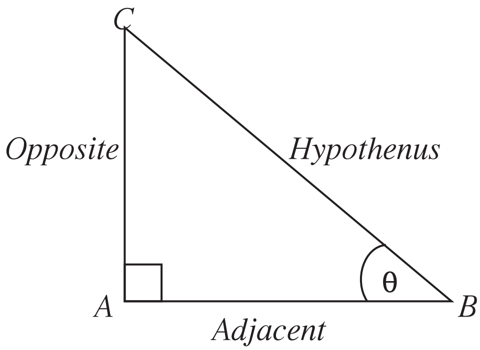
Core Competencies
- Critical Thinking and Problem Solving
- Personal Development and Leadership

Lesson Presentation
Identify the parts of a circle



Circumference, in geometry is the perimeter of a circle or ellipse. That is, the circumference would be the arc length of the circle as if it were opened up and straightened out to a line segment.

(Check www.wikipedia.org for further reading)

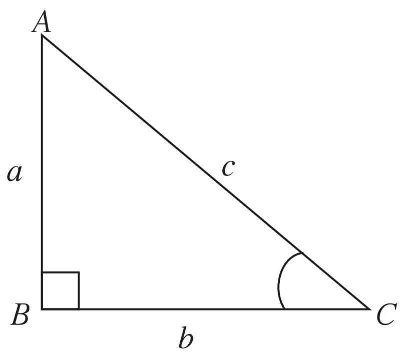


$\triangle ABC$ is a right-angled triangle.

Let learners identify the parts of the right-angled triangle as labelled above. Use $\triangle ABC$ to explain the trigonometric ratios as in sine, cosine and tangent to learners.

$$SOH \longrightarrow \sin^\circ = \frac{opposite}{hypotenuse}$$
$$CAH \longrightarrow \cos^\circ = \frac{(adjacent)}{hypotenuse}$$
$$TOA \longrightarrow \tan^\circ = \frac{(opposite)}{adjacent}$$

Pythagoras' Theorem



$$c^2 = a^2 + b^2$$

Lead and guide learners to state the Pythagoras' theorem. In a right-angled triangle, the square of the hypotenuse is equal to the sum of the square of the opposite and the adjacent. *Refer to page 209 of Learner's Book.*

Lead and guide learners to solve questions as in exercise learners to solve the questions as in exercise 2 at *page 213 of Learner's Book 8*. Let learners present and discuss their solutions.

Applications of Pythagoras theorem

Refer to page 213–218 of Learner's Book 8.

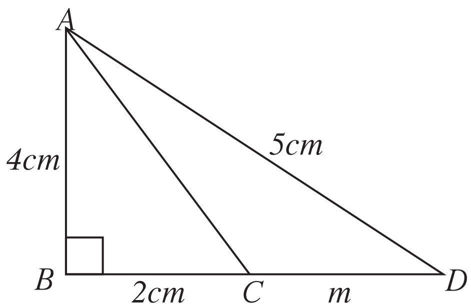
Discuss with learners some of the applications of the Pythagoras theorem in real life. Some of the real-life applications of Pythagoras theorem are;

1. It is used in construction and architecture
2. It is used in two-dimensional navigation to find the shortest distance
3. It is used to survey the steepness of the slopes of mountains or hills
4. It is used to calculate the length of the longest item to be kept in a room
5. It is used to determine the height and measurements in the construction sites

Lead learners to solve problems involving application of Pythagoras theorem

Diagnostic Assessment

1. Find the value of m in the diagram below



Solution

Consider $\triangle ABD$

$$|AD|^2 = |AB|^2 + |BD|^2$$
$$5^2 = 4^2 + (2 + m)^2$$
$$25 = 16 + 4 + 4m + m^2$$
$$25 = 20 + 4m + m^2$$
$$25 - 20 = 4m + m^2$$
$$5 = 4m + m^2$$

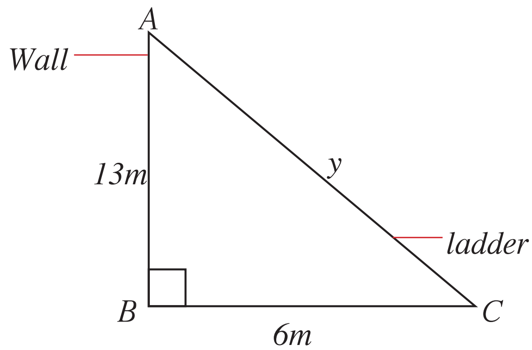
$$m^2 + 4m - 5 = 0$$
$$(m^2 - m) + (5m - 5) = 0$$
$$m(m - 1) + 5(m - 1) = 0$$
$$(m - 1)(m + 5) = 0$$
$$m - 1 = 0 \Rightarrow m = 1$$
$$or\ m + 5 = 0 \Rightarrow m = -5$$

Hence $m = 1cm$

2. A ladder leans against a vertical wall of height $13m$. If the foot of the ladder is $6m$ away from the wall, calculate the length of the ladder.

Solution

NB: To solve application problems of this nature one needs to make a sketch of the problem to guide you to solve the problem.



Length of the ladder $|AC| = y$

Using the Pythagoras' theorem, we have,

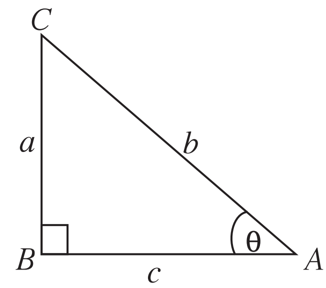
$$y^2 = (13)^2 + 6^2$$
$$y^2 = 169 + 36 = 205$$
$$y = \sqrt{205} = 14.378$$
$$y = 14.3$$

Therefore the length of the ladder is $14.3m$

Basic Trigonometric Ratio involving Right-Angled Triangle

Refer to page 219-221 of Learner's Book 8.

Lead and guide learners to use the acronyms, SOH, CAH, TOA.



In the right-angled triangle above, the side $|AB|$ is the adjacent, $|BC|$ is the opposite and $|AC|$ is the hypotenuse.

$$SOH \Rightarrow sine = \frac{opposite}{hypothenus} = \frac{|BC|}{|AC|}$$

$$\therefore \sin\theta = \frac{a}{b}$$

$$CAH \Rightarrow cosine = \frac{adjacent}{hypothenus} = \frac{|AB|}{|AC|}$$

$$\therefore Cos\,\theta = \frac{c}{b}$$

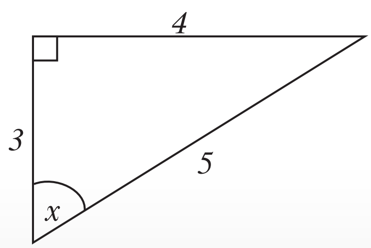
$$TOA \Rightarrow tangent = \frac{opposite}{adjacent} = \frac{|BC|}{|AB|}$$

$$\therefore tan\,\theta = \frac{a}{c}$$

Involve learners in solving other examples.

Diagnostic Assessment

1.



Use the diagram above to find $\sin x, \cos x \; and \tan x$

Solution:

$$\sin x \Rightarrow SOH = \frac{Opposite}{Hypotenuse}$$

$$\sin x = \frac{4}{5}$$

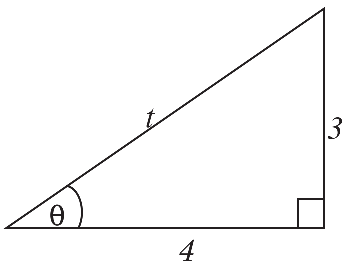
$$\cos x \Rightarrow CAH = \frac{Adjacent}{Hypotenuse}$$

$$\cos x = \frac{3}{5}$$

$$\tan x \Rightarrow TOA = \frac{Opposite}{Adjacent}$$

$$\tan x = \frac{4}{3}$$

2.



From the diagram above, Find;

i. The value of t
ii. $\cos\theta$
iii. $\sin\theta$
iv. $\tan\theta$

Solution

i. To find t we need to apply the Pythagoras' theorem.

$$t^2 = 4^2 + 3^2$$
$$t^2 = 16 + 9$$
$$t^2 = 25$$
$$t = \sqrt{25} = 5$$

i. $\cos\theta = \frac{4}{5}$

ii. $\sin\theta = \frac{3}{5}$

iii. $\tan\theta = \frac{3}{4}$

Angle of Elevation and Angle of Depression *(Refer to page 222 of Learner's Book 8)*

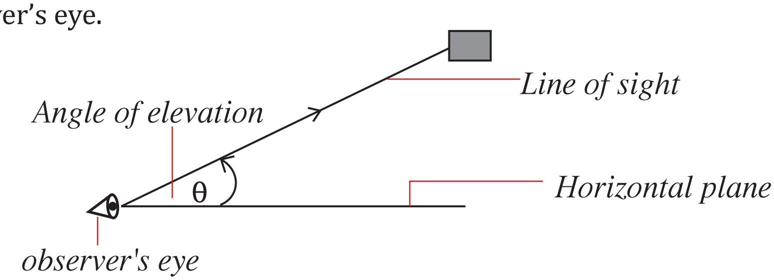
- **Lesson Presentation**

Lead learners to explain what is meant by elevation and depression as applied to Mathematics.

Angle of Elevation

Elevation simply means the height to which something is raised.

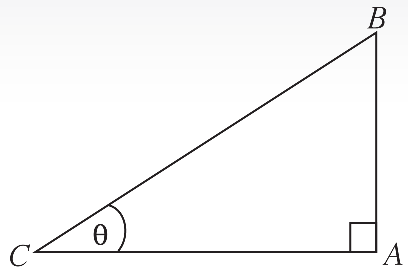
Angle of elevation is defined as an angle between the horizontal plane and the line of sight from the observer's eye.



For instance, an observer is looking at a bird that is sitting at the rooftop of a building, then there is an angle of elevation formed, which is inclined towards the bird from the observer's eye.

This angle of elevation is used to find distances, heights of buildings or towers with the help of trigonometric ratios.

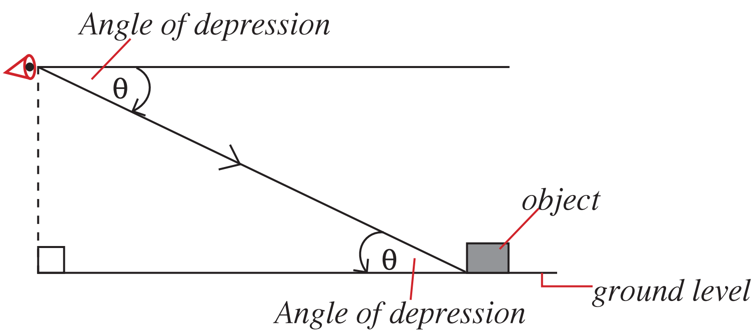
From the figure above, we can see that the observer's eye is looking at the object while standing on the line horizontal ground, forming an angle $\theta$with the horizontal line. If we join an imaginary line between the object (bird) and the end of the horizontal line, we will form a right-angled triangle.



We can use the concept of trigonometry (i.e. trigonometric ratios) to find the distance of the observer from the bird. We can also find the height from the object (bird) to the horizontal line.

Angle of depression
Angle of depression is the angle between the horizontal line of sight and the line of sight down to an object.

For example, if you were standing on top of a hill or a building, looking down at an object, you could measure the angle of depression



The angle alternates, hence we can use the angle formed between the eye sight looking down and the ground level. See the diagram above.

Let learners solve questions at exercise 6 and present their solutions to the class.
Refer to page 224 of Learner's Book 8.

Addition and Subtraction of Vectors
Refer to page 228-229 of Learner's Book 8.

Lesson Presentation

Explain what is column vectors to learners. A column vector is simply a vector whose components are listed vertically in a single column. Thus, $\begin{pmatrix} x \\ y \end{pmatrix}$.

Also, a column vector is a matrix with one column. We can add vectors in column form. For example, given $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} m \\ n \end{pmatrix}$ *then* $\overrightarrow{AB} + \overrightarrow{BC}$ can be given as

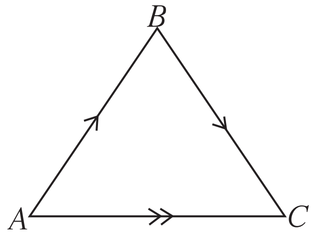
$$\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} x + m \\ y + n \end{pmatrix}$$

Here, we add the $x - component$ and $y - component$ respectively.

Similarly, we can subtract vectors in component form. Hence $\overrightarrow{AB} - \overrightarrow{BC}$ is

$$\overrightarrow{AB} - \overrightarrow{BC} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} x - m \\ y - n \end{pmatrix}$$

When adding vectors in directional form on a diagram, we need to follow the arrow directions to add them accurately.



Study the diagram above. The arrow direction from vertex A to B can be written in vector form as $\overrightarrow{AB}$. The direction from vertex B to C can be written as $\overrightarrow{BC}$ and from A to C can be written as $\overrightarrow{AC}$. Note that the side from A to C has a double arrow on it. This means that it is the resultant of the two vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$

By addition of vectors, we have;

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

This is known as triangular law of adding vectors. Lead and guide learners to add vectors in column form.

Example 1.

$$\overrightarrow{AB} = \begin{pmatrix}3\\4\end{pmatrix} \text{ and } \overrightarrow{BC} = \begin{pmatrix}7\\12\end{pmatrix}$$

Find;

i. $\overrightarrow{AB} + \overrightarrow{BC}$

ii. $\overrightarrow{BC} - \overrightarrow{AB}$

Solution

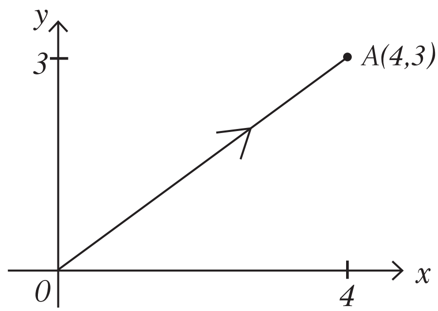
i. $\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}7\\12\end{pmatrix} = \begin{pmatrix}3+7\\4+12\end{pmatrix} = \begin{pmatrix}10\\16\end{pmatrix}$

ii. $\overrightarrow{BC} - \overrightarrow{AB} = \begin{pmatrix}7\\12\end{pmatrix} - \begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}7-3\\12-4\end{pmatrix} = \begin{pmatrix}4\\8\end{pmatrix}$

Position Vector

A position vector is defined as a vector that symbolises either the position or the location of any given point with respect to any arbitrary reference point ;like the origin.

The direction of the position vector always points from the origin of that vector towards a given point.
For example, the position vector of an object is measured from the origin. In general,



The position of the point $A(4,3)$ as shown on the plane above is $\overrightarrow{OB} = \begin{pmatrix}4\\3\end{pmatrix}$

Refer to page 230 of Learner's Book 8.
A vector that is written in the form $\overrightarrow{AB}$ means that it has an initial point of A and a final point of B. $\overrightarrow{AB}$ can be expressed in position vector form as, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

It is important to note that we can use small letters to represent vectors.

For instance, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ can be written as $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$, where b and a are position vectors of the points A and B.

Diagnostic Assessment

1. Given that $\boldsymbol{m} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ *and* $\boldsymbol{n} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$, calculate t such that $\boldsymbol{t} = \boldsymbol{n} + \boldsymbol{m}$

Solution

$$\boldsymbol{t} = \boldsymbol{n} + \boldsymbol{m}$$
$$= \begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$
$$t = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$$

2. Given that $\boldsymbol{a} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$ *and* $b = \begin{pmatrix} -1 \\ 12 \end{pmatrix}$, Find $\boldsymbol{a} + \boldsymbol{b}$

Solution

$$a + b = \begin{pmatrix} 0 \\ -5 \end{pmatrix} + \begin{pmatrix} -1 \\ 12 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

Lead and guide learners to solve further questions. *Refer to page 250 of Learner's Book 8.*

Subtraction of Vectors *(Refer to page 228 – 229 of Learner's Book 8)*
Just like addition of vectors, we can subtract vectors in component form.

For example, given $a = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $b = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Calculate $\boldsymbol{a} - \boldsymbol{b}$

Solution

$$\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 3 - -1 \\ -5 - 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$$

Guide learners to be very careful about the minus signs when doing subtraction of vectors.

Equality of Vectors *(Refer to page 233-235 of Learner's Book 8)*
When two given vectors are equal, we equate their corresponding components. Thus; if
$\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ *and* $\boldsymbol{b} = \begin{pmatrix} m \\ n \end{pmatrix}$ *and* $a = b$ then, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix} \Rightarrow x = m$ *and* $y = n$

Example 1. Find the value of y if $\begin{pmatrix} y - 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 19 \\ 3 \end{pmatrix}$

Equating corresponding components, we have

$$y - 1 = 19$$
$$y = 19 + 1$$
$$y = 20$$

Multiplication of a Vector by a Scalar

Refer to pages 230 – 232 of Learner's Book 8.

When a vector is multiplied by a scaler quantity then the magnitude of the vector changes in accordance with the magnitude of the scalar but the direction of the vector remains unchanged.

For instance, if the vector $\overrightarrow{\mathbf{AB}} = \begin{pmatrix} x \\ y \end{pmatrix}$ and k is the scaler multiple, then kAB will be $k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$.

Diagnostic Assessment

1. Given that $\boldsymbol{m} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ Calculate

   a. $2\boldsymbol{m} + \boldsymbol{n}$
   b. $3\boldsymbol{n} - \boldsymbol{m}$
   c. $3\boldsymbol{m} - 4\boldsymbol{n}$

Solutions:

a. $2\boldsymbol{m} + \boldsymbol{n} = 2\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$

b. $3\boldsymbol{n} - \boldsymbol{m} = 3\begin{pmatrix} 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$

c. $3\boldsymbol{m} - 4\boldsymbol{n} = 3\begin{pmatrix} 2 \\ 3 \end{pmatrix} - 4\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$

Involve learners to solve the questions.

Sub-Strand/Unit 3: Position And Transformation

Refer to pages 236 – 241 of Learner's Book 8.

Content Standards
B8.3.3.1: Perform a single transformation (ie. Rotation) on a 2D shape using graph paper (including technology) and describe the properties of the image under the transformation (i.e. congruence)

Learning expectations:
After studying this sub-strand/unit, the learner will be able to:
- identify examples of rotation situation in everyday life
- rotate a shape through a given centre of rotation and angle of rotation using rotation rules
- determine the angle of rotation using point of an object
- use multiple and varied examples of rotation on coordinate planes

Keywords: *Refer to Learner's Book 8, page 236.*
Guide learners to use the their dictionaries to find the contextual meaning of the keywords.
- *rotation, clockwise, anti-clockwise, coordinate, angle, congruent shapes*

Core Competencies
- Critical Thinking and Problem Solving
- Creativity and Innovation
- Digital Literacy

Lesson Presentation
Lead and guide learners to define what is rotation. Rotation is a type of transformation that take each point in a figure and rotates it in a certain number of degrees around a given point. Rotation is defined as the motion of an object around a centre or an axis. The result of a rotation is a new figure called the image. The image is congruent to the original figure.

Although a figure can be rotated any number of degrees, the rotation will usually be a common angle such as $45^\circ, 90^\circ, 180^\circ$ *or* 270°.

Rotation can be in two directions:
1. Clockwise rotation and
2. Anti-clockwise rotation

Guide learners to use graph to practice the activity at *page 240 Learner's Book 8.* Let learners solve the question at exercise 1 and present their solutions to the class.

STRAND/CHAPTER 4: HANDLING DATA

Sub-Strand/Unit 1: Collecting And Classifying Data

Refer to page 242-259 of Learner's Book 8.

Content Standards
B8.4.1.1: Select, justify, and use appropriate methods to collect data (quantitative and qualitative), and use the data (grouped/ungrouped) to construct and interpret frequency tables, bar charts, pie charts, and pictograms to solve and/or pose problems.

B8.4.1.2: Demonstrate an understanding of measures of central tendency (mean, median, mode) and range for grouped data and explain when it's most appropriate to use the mean, median, or mode.

Learning Expectations:
After studying this sub-strand/unit, the learner will be able to;
- identify types of given data including numerical, categorical, grouped and ungrouped data
- select and justify a method to collect data (qualitative and quantitative) to answer a given question
- organise data (grouped or ungrouped) present it in frequency tables, line graphs, pie charts, bar graphs and pictographs (representations include infographics, waffle diagrams, box and whisker plots and steam and lead plots) and analyse it to solve and/or prose problems.
- calculate the mean, median, and mode for a given set of ungrouped data and explain why those values may be same or different
- justify a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings.

Keywords: *Refer to Learner's Book 8, page 242.*
Lead and guide learners to discuss the following words or terminologies.
- *data, categorical, quantitative, qualitative, frequency, mean, median , mode, range, central tendency*

Core Competencies
- Critical Thinking and Problem Solving
- Communication and Collaboration
- Digital Literacy
- Personal Development and Leadership

Types of Data *(Refer to page 243 of Learner's Book 8)*

Data in Mathematics is a collection of facts and figures that can be in any form, numerical or non-numerical.

1. Numerical data is the type of data that you can calculate and it is in number form such as scores of students in a class test, wages of workers in an organizations, height of footballers in a team, etc.
2. Non-numerical data is the data that can be collected only and not be calculated. Examples are characteristics of a person, the physical appearance of an animal or other observations not including numbers. Other examples include; the taste of a fruit, the color of a crayon, etc.
3. Qualitative Data: Qualitative Data does not have a numerical value but it contains various characteristics about objects or people such as; color, shape, taste, etc
4. Quantitative data: Quantitative data is a numerical data that includes information like age, time, height, weight, marks or score etc. The data is collected by measuring it on a required parameters. For instance, data on the number of students playing different sports from your class in in quantitative form.
5. Categorical data is a collection of information that is divided into groups.

Let learners do further search and present their findings on some of the terminologies stated earlier.

Categorical Data

Put learners in groups of five to do the activities at examples 1 and 2 at *page 244 of Learner's Book 8.*

Let the groups present their solutions to the class. Also let the groups do exercise 2 and present their solutions to the class.

What is frequency?
Frequency refers to the number of times an event or a value occurs.

What is frequency table?
A frequency table that lists items and shows the number of times the items occur. It is a method used to organize the data given so that it makes it more meaningful and easier to understand.

Lead learners to study the frequency table presented under grouped frequency table at *page 233 of Learner's Book.* Let learners do exercise 3 and discuss their solutions among themselves.

Pie Chart *(Refer to pages 247 of Learner's Book 8)*

What is a pie chart?
A pie chart is a circular graph of information that is divided into various sectors to compare the distribution of data or statistics.

There are various applications of pie chart some of the applications of pie chart can be found in business, school and at home.

For business, pie chart can be used to show the success or failure of certain products or services . They can also be used to show market reach of a business compared to similar businesses.
Lead and guide learners on how to draw a pie using the information given on *page 247, example 1 of Learner's Book 8.*

The waffle diagram *(Refer to page 248 of Learner's Book 8)*

A waffle chart is essentially a square display made up of 100 smaller squares organized in a 10-by-10 gride where each box corresponds to 1%.

The colored boxes represent the percentage of the target that was met with 100 percent being the entire goal.

Let learners practice how to draw a waffle diagram using the example 1 at page 236 of Learner's Book 8. You can search www.thedataschool.co.uk to practice how to draw the waffle chart using the MS excel on the computer.

Stem and Leaf plot, Line graphs, Pictogram
Refer to pages 250-252 of Learner's Book 8.

Stem and Leaf plot
A stem and leaf plot is a technique used to classify either discrete or continuous variables.
A stem and leaf plot is used to organise data as they are collected. A stem and leaf plot looks like a bar graph. Each number in the data is broken down into a stem and leaf, The stem of the number includes all but the last digit. The leaf of the number will always be a single digit.

Elements of a good stem and leaf plot
A good stem and leaf plot shows the following;
1. Shows the first digit of the number (thousands, hundreds or tens) as the stem and shows the last digit (ones) as the leaf.
2. Usually uses whole numbers. Anything that has a decimal point is rounded to the nearest whole number.
3. Looks like a bar graph when it is turned on its side.
4. Shows how the data are spread, that is highest number, lowest number, most common number and outliers (a number that lies outside the main group of numbers)

How to draw a stem and leaf plot
1. On the left hand side of the page, write down the thousands, hundreds , or tens (all digits but the last one) Those will be your stems.
2. Draw a line to the right of these stems
3. On the other side of the line, write down the ones (the last digit of a number). These will be your leaves.

Diagnostic Assessment

Example:
A teacher asked 10 of his learners how many books they had read in the last 12 months. Their responses were as follows: *12,23,19,6,10,7,15,25,21,12*

Prepare a stem and leaf plot for the data.

Solution
Note that the number 6 can be written as 06 and 7 as 07. This means that they have a stem of 0 and a leaf of 6 and 7 respectively.

Stem and leaf plot of books read by 10 learners in 12 months

Stem	**Leaf**				
0	6		7		
1	2	9	0	5	2
2	3	5		1	

1. Stem 0 represents the class interval 0 to 9
2. Stem 1 represents the class interval 10 to 19 and
3. Stem 2 represents the class 20 to 29

Usually, a stem and leaf plot is ordered, which is ordered, which simply means the leaves are arranged in ascending order from left to right. Also, there is no need to separate the leaves (digits) with punctuation marks (commas or periods) since each lead is always a single digit.

Hence, in ascending order the stem and leaf plot

Stem	**Leaf**
0	6 7
1	0 2 2 5 9
2	1 3 5

Refer to pages 250–251 of Learner's Book 8 and Let learners do the exercises in graphs.

Pictograph

Refer to pages 252–253 of Learner's Book.
Lead and guide learners to draw the pictograph on page 240 of the Learner's Book. Let learners work in groups and discuss the solutions among themselves.

Measure of central tendency *(Refer to page 254 of Learner's Book 8)*
A measure of central tendency (also referred to as measure of centre or central location) is a summary measure that attempts to describe a whole set of data with a single value that represents the middle or centre of its distribution.

Lead and guide learners to define the measures of central tendency; mean, mode and median.

Let learners work the given examples and discuss the solutions among themselves. Refer to *pages 254–259 of Learner's Book 8.*

Sub-Strand/Unit 2: Chance or Probability

Referto pages 260-266 of Learner's Book 8.

Content Standards
B8.4.2.1: Identify the sample space for a probability experiment involving two independent events and express the probabilities of given events as fractions. Decimals, percentages and/or ratios to solve problems.

Learning Expectations:
After studying this sub-strand/unit, the learner will be able to:
- perform a probability experiment involving two independent events such as drawing coloured bottle tops from a bag with replacement and list the elements of the sample.
- express the probabilities of the events as fractions, decimals, percentages and/or ratios. For example by using a tree diagram, table or other graphic organiser.

Keywords: *Refer to Learner's Book 8, page 260.*
Guide learners to use the their dictionaries to find the contextual meaning of the keywords.
- *probability, chance independent, tree diagram replacement*

Core Competencies
- Critical Thinking and Problem Solving
- Communication and Collaboration
- Cultural Identity and Global Citizenship
- Personal Development and Leadership

What is probability?
Probability of a number that reflects the chance of likelihood that a particular event will occur. Probabilities can be expressed as proportions that range from 0 to 1 and they can also be expressed as percentages ranging from 0 to 100%.

Independent Events *(Referto page 261 of Learner's Book 8)*
Two events are independent if the occurrence of the event does not affect the chances of the occurrence of the other event. The mathematical formulation of the independence of events A and B is the probability of the occurrence of both

A and B being equal to the product if the probabilities of A and B.
This is expressed as $p(A \text{ and } B) = p(A \cap B) = P(A) \times P(B)$

Lead and guide learners to solve the given examples.

Tree Diagrams *(Refer to pages 263–266 of Learner's Book 8)*
A tree diagram is a method in the field of general mathematics, probability and statistics that helps to calculate the number of possible outcomes of an event or problem and to note those potential outcomes in an organised way. Tree diagrams display all the possible outcomes of an event. Each branch in a tree diagram represents a possible outcome. Tree diagrams can be used to find the number of possible outcomes and calculate the probability of possible outcomes.

Lead and guide learners to solve the given examples and exercises given.
You may search for further reading of *nrich.math.org* and *byjus.com* on the internet.

ANSWERS TO THE LEARNER'S BOOK EXERCISES

STRAND/CHAPTER 1: NUMBER

SUB-STRAND/UNIT 1: NUMBER AND NUMERATION SYSTEM

Exercise 1 *(Refer to page 3 of Learner's Book 8)*

1. Nine billion, five million, seven thousand and four.
2. Six-hundred and ninety-four billion, two hundred and thirty-one million, six hundred and twenty-three thousand, four hundred and thirty-nine.
3. Twenty-five billion, seven hundred and eight million, three hundred and ninety-seven thousand, nine-hundred and thirty-six
4. Four billion, five hundred and fifty-six million, seven hundred and eighty-nine thousand, four hundred and fifty-six
5. Thirty-three million, four hundred and seventy-eight thousand, nine hundred and ten.
6. Four thousand and twenty
7. Sixty-eight
8. Four million, two hundred and eighty-nine thousand, one hundred
9. Two billion and four
10. Fifty-five billion, six hundred and sixty-six million, seven hundred and seventy-seven thousand, eight hundred and eighty-one.

Exercise 2 *(Refer to page 5 of Learner's Book 8)*

1. 2,000,040
2. 45,000,300,010
3. 721,004,003,009
4. 89,200
5. 69,000,000,029

Exercise 3 *(Refer to pages 6-7 of Learner's Book 8)*

1. a. 5,000,000   5,100,200   5,200,400
   b. 5,000,000   5,300,000   5,600,000
   c. 5,000,000   5,400,500   5,801,000
   d. 5,000,000   5,007,002   5,014,004
   e. 5,000,000   5,500,000   6,000,000

Exercise 4 *(Refer to page 9 of Learner's Book 8)*

1. $<$
2. $>$
3. $<$
4. $<$
5. $>$
6. $<$
7. $>$
8. $=$
9. $>$

Exercise 5 *(Refer to page 9 of Learner's Book 8)*

1. $<$
2. $<$
3. $<$
4. $<$
5. $<$

Exercise 6 *(Refer to page 16 of Learner's Book 8)*

1. 4.56789×10^{5}
2. 1.0×10^{9}
3. 8.7345678×10^{7}
4. $6.789456203 \times 10^{9}$
5. 2.00300400×10^{8}
6. 9.0×10^{9}
7. 8.023000×10^{6}
8. 1.2456789×10^{7}
9. 4.734×10^{3}
10. 4.2×10^{9}

Exercise 7 *(Refer to page 18 of Learner's Book 8)*

1. 6800
2. 40340
3. 94600
4. 30040
5. 43000

Exercise 8 *(Refer to page 21 of Learner's Book 8)*

1. $GH¢6,000.00$
2. $GH¢60,000.00$
3. 325 marks

Exercise 9 *(Refer to page 27 of Learner's Book 8)*

1. 2
2. 3
3. 4
4. 13
5. 15
6. 25
7. 12
8. 24
9. 18
10. 11

Exercise 10 *(Refer to page 46 of Learner's Book 8)*

1. a $\{1,2,3,4,6,12\}$
   b $\{1,2,3,4,6,8,12,24\}$
   c $\{1,2,4,5,10,20\}$
   d $\{1,2,3,4,6,8,9,12,18,24,36,72\}$
   e $\{1,3,5,9,15,45\}$

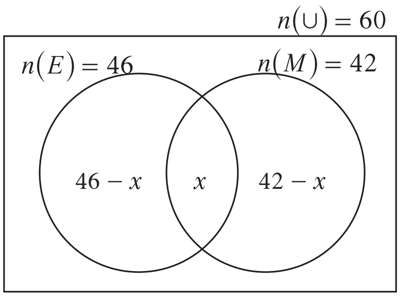
2. a. 1 b. 1, 3 c. 1, 3 d. 1, 3 e. 1, 3

Exercise 11 *(Refer to page 48 of Learner's Book 8)*

1. $\{1,3\}$
2. ϕ
3. $\{1,2,3,4,5,6,7,9\}$

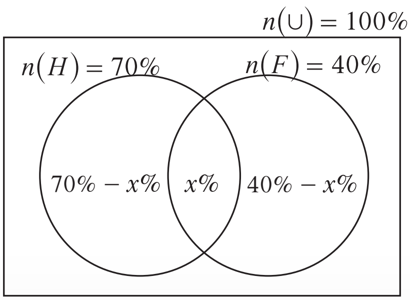
Exercise 12 *(Refer to page 42 of Learner's Book 8)*

1.
   i.



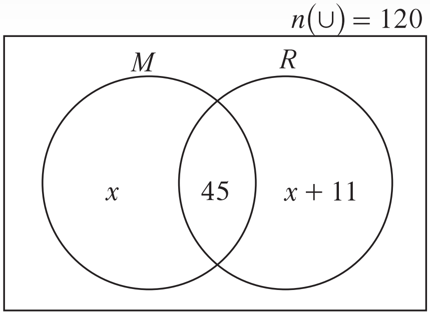
   ii. 28 students passed both subject
   iii. 32 students passed only one subject

2.



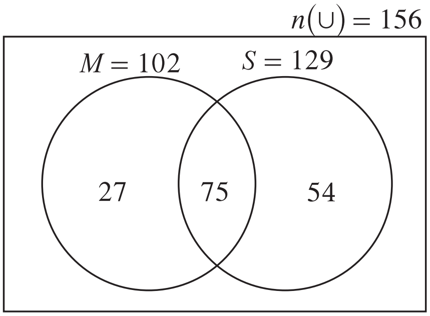
10% of the pupil's study both subjects.

3.



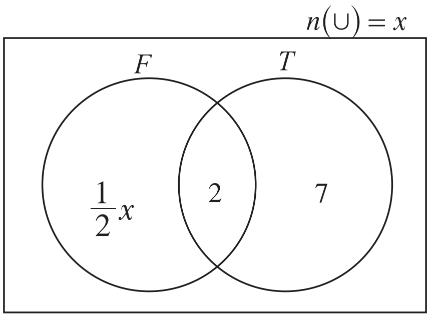
ii. 32 pupils passed in mathematics only.
iii. 43 pupils passed in RME

4. a) 27 pupils passed mathematics
b) 54 pupils passed science.



5.

i.



ii. 18 students are in the sporting club
iii. 9 students play football only.

Exercise 13 *(Refer to page 45 of Learner's Book 8)*

1. 199679
2. 273951
3. 5311
4. 1109114
5. 848474

Exercise 14 *(Refer to page 46 of Learner's Book 8)*

1. 12217.28
2. 10021.73
3. 3781.53

Exercise 15 *(Refer to page 52 of Learner's Book 8)*

1. 48250
2. 268920
3. 119799
4. 193880

Exercise 16 *(Refer to page 58 of Learner's Book 8)*

1. $x = -1$
2. $x = 4$
3. $\frac{125}{27}$
4. 4^5 *or* 1024
5. 75^3

Exercise 17 *(Refer to page 60 of Learner's Book 8)*

1. $x = 0, y = 0$
2. $x = 6, y = -1$

STRAND/CHAPTER 1: NUMBER
SUB-STRAND/UNIT 2: NUMBER OPERATIONS

Exercise 1 *(Refer to page 63 of Learner's Book 8)*

1. 132
2. 5
3. 8
4. 72
5. 126
6. 4
7. 4
8. 7
9. 12
10. 12
11. 108

Exercise 2 *(Refer to page 67 of Learner's Book 8)*

A. 1. 0.8, 80%
2. 0.875, 87.5%
3. 0.375, 37.5%
4. 0.3, 30%
5. 0.30769, 30.77%

Exercise 3 *(Refer to page 67 of Learner's Book 8)*

A. 1. 0.75 2. 0.80 3. 0.95 4. 0.375 5. 0.625

B. 6. $\frac{17}{20}$ 7. $\frac{13}{20}$ 8. $\frac{3}{10}$ 9. $\frac{9}{20}$ 10. $\frac{1}{20}$

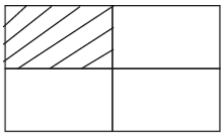
Exercise 4 *(Refer to page 68 of Learner's Book 8)*

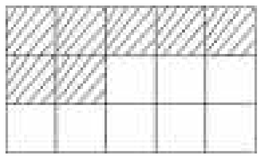
1. 101
2. 120050
3. 2438000
4. 720
5. 5780
6. 87800

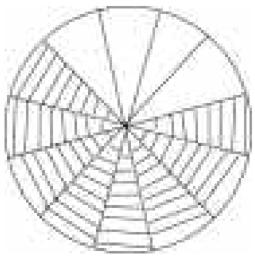
STRAND/CHAPTER 1: NUMBER
SUB-STRAND/UNIT 3: FRACTIONS, DECIMALS AND PERCENTAGES

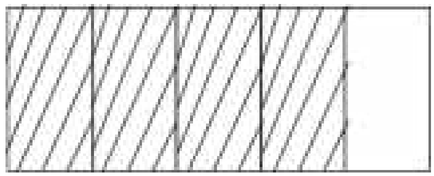
Exercise 1 *(Refer to pages 74 - 75 of Learner's Book 8)*

1.

a. 

b. 

c. 

d. 

2. a. $\frac{5}{12}$ b. $\frac{6}{12}$ c. $\frac{3}{6}$ d. $\frac{3}{8}$

3. a. $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}$ b. $\frac{8}{13}, \frac{16}{26}, \frac{24}{39}, \frac{40}{65}$ c. $\frac{9}{23}, \frac{18}{46}, \frac{27}{69}, \frac{36}{92}$

4. a. $\frac{27}{32}$ b. $\frac{2}{3}$ c. $\frac{1}{3}$ d. $\frac{1}{2}$

5. a. $1\frac{8}{13}$ b. $8\frac{1}{15}$ c. $2\frac{15}{17}$ d. $4\frac{2}{21}$

6. a. $\frac{86}{11}$ b. $\frac{59}{5}$ c. $\frac{95}{3}$ d. $\frac{5}{2}$

Exercise 2 *(Refer to page 84 of Learner's Book 8)*

1. $\frac{17}{14}$ *or* $1\frac{3}{14}$
2. $\frac{133}{75}$ *or* $1\frac{58}{75}$
3. $\frac{92}{63}$ *or* $1\frac{29}{63}$
4. $\frac{3}{2}$ *or* $1\frac{1}{2}$
5. $\frac{51}{41}$ *or* $1\frac{10}{41}$

Task learners to answer the rest of the questions. Check and mark learners anwers.

Exercise 3 *(Refer to page 86 of Learner's Book 8)*

1. 26
2. 9
3. − 39
4. 49
5. 74
6. −4
7. 2
8. 63
9. 20
10. 13

Exercise 4 *(Refer to page 88 of Learner's Book 8)*

1. a. $\frac{13}{12}$ *or* $1\frac{1}{12}$ b. 0 c. $\frac{149}{30}$ *or* $4\frac{29}{30}$ d. $\frac{88}{25}$ e. $-\frac{4}{3}$ f. $\frac{93}{20}$ g. 1

Exercise 5 *(Refer to page 90 of Learner's Book 8)*

1. $\frac{13}{12}$ *or* $1\frac{1}{12}$
2. $\frac{3}{2}$ *or* $1\frac{1}{2}$
3. $\frac{6}{11}$
4. $\frac{-177}{40}$
5. $\frac{2}{21}$
6. $\frac{13}{30}$

Exercise 6 *(Refer to page 92 of Learner's Book 8)*

1. $Perimeter = 10\frac{1}{6}cm$ $area = 5cm^2$
2. 70%
3. $\frac{2}{15}$
4. Esi's drink should taste stronger of orange than Fusena's drink.
5. $\frac{11}{21}$

STRAND/CHAPTER 1: NUMBER
SUB-STRAND/UNIT 4: NUMBER RATIO AND PROPORTION

Exercise 1 *(Refer to page 98 of Learner's Book 8)*

1. 5,234,000
2. 90000 seconds
3. 2,100 pesewas
4. 1,240 millimetres
5. 60.34 metres
6. 434 metres

Exercise 2 *(Refer to page 100 of Learner's Book 8)*

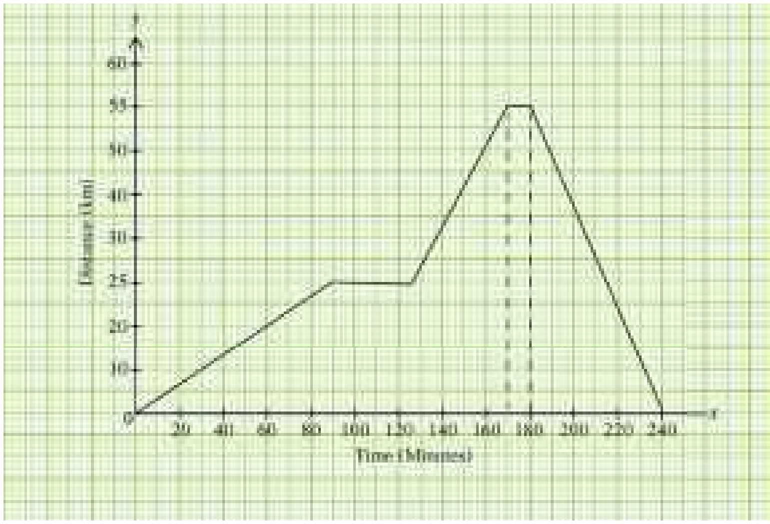
1. Adwoa walks faster than Kofi
2. 65minutes (1hr 5mins)
3. Akosua's vehicle (find the speed of both vehicles Ama 14.17k/h; Akosua 18.57km/h

Exercise 3 *(Refer to page 100 of Learner's Book 8)*

1. 20 lawns
2. 200 calories per hour.
3. 5 glasses per minute
4. *GH¢*9.00 per hour
5. 15 toys per hour

Exercise 4 *(Refer to page 105 of Learner's Book 8)*

1. i. 50mins
   ii. 30km
   iii. 45km/h
   iv. 55km/h
   v. 116.67km/h



Exercise 5 *(Refer to page 107-109 of Learner's Book 8)*

1. Non-proportional
2. Proportional
3. Non-proportional
4. Proportional
5. Proportional
6. Non-proportional
7. Non-proportional
8. Proportional

Exercise 6 *(Refer to page 112 of Learner's Book 8)*

1. 2
2. 2
3. 50
4. 5
5. 5

STRAND/CHAPTER 2: ALGEBRA
SUB-STRAND/UNIT 1: PATTERNS AND RELATIONS

Exercise 1 *(Refer to page 121 of Learner's Book 8)*

1. $x = -6$
2. $y = \frac{29}{3}$

Exercise 2 *(Refer to page 125 of Learner's Book 8)*

1. 3
2. -2
3. $\frac{11}{8}$
4. 3
5. -3
6. $\frac{1}{3}$
7. -2
8. $\frac{-5}{3}$

Exercise 3 *(Refer to page 126 of Learner's Book 8)*

1. -3
2. -5
3. $\frac{1}{2}$
4. $\frac{1}{3}$
5. $\frac{3}{2}$
6. $-\frac{1}{7}$
7. -5
8. 2

Exercise 4 *(Refer to page 129 of Learner's Book 8)*

1. $y = -\frac{1}{3}x + 5$
2. $y = x + 1$
3. $y = -2x - 3$
   $y = -7$

Exercise 5 *(Refer to page 133 of Learner's Book 8)*

1. a. $y = -5$
   b. $y = -3x - 1$
   c. $y = 2x + 5$
   d. $y = \frac{1}{4}x + 8$

2. a. $y = \frac{2}{7}x - \frac{22}{7}$
   b. $y = 3x - 1$
   c. $y = \frac{2}{3}x + \frac{8}{3}$
   d. $y = \frac{3}{2}x + 1$

Exercise 8 *(Refer to page 139 of Learner's Book 8)*

1.

Cost $(GH¢)$	20	40	60	80	100
Number of Oranges	5	10	15	20	25

2. i. $GH¢4.00$
   ii. 30 oranges
   iii. $GH¢1,728.00$
   iv. 35 oranges

STRAND/CHAPTER 2: ALGEBRA
SUB-STRAND/UNIT 2: ALGEBRAIC EXPRESSIONS

Exercise 1 *(Refer to page 142 of Learner's Book 8)*

1. $8 - 2x$
2. $10x - 28$
3. $2x^2 - 7x + 14$
4. $2x + 2y - 2z + 2k$
5. $x - a + 3bx - 3ab$

Exercise 2 *(Refer to page 146 of Learner's Book 8)*

1. $x^2 + x - 6$
2. $y^2 - 5y + 6$
3. $2a^2 - ab - b^2$
4. $a^2 - b^2$
5. $m^2 - 2mn + n^2$

Exercise 3 *(Refer to page 149 of Learner's Book 8)*

1. $\frac{4a}{15by}$
2. 2
3. $\frac{1}{6}$
4. $\frac{3b}{2}$
5. 1

Exercise 4 *(Refer to page 153 of Learner's Book 8)*

a. $\frac{5x}{9}$

b. $\frac{a+3b}{6}$

c. $\frac{-a-3b}{6}$

d. $\frac{-x-21}{12}$

e. $\frac{y+x}{4a}$

Exercise 5 *(Refer to page 157 of Learner's Book 8)*

1. 664
2. 1
3. $\frac{-2}{5}$
4. -5

Exercise 6 *(Refer to page 161 of Learner's Book 8)*

1. $16x(1+4xy)$
2. $P(7p+1)$
3. $2ab(2c-1)$
4. $a(1-4x)+b(4x-3)$
5. $a^2(x-y)+b^2(y-1)$
6. $(t-3c)(p+2q)$

STRAND/CHAPTER 2: ALGEBRA
SUB-TRAND/ UNIT 3: VARIABLES AND EQUATIONS

Exercise 1 *(Refer to page 164 of Learner's Book 8)*

i. $2 + 2x < 4$
ii. $4x < 2x$
iii. $x \geq 21$

Exercise 2 *(Refer to page 168 of Learner's Book 8)*

1. $\{x : x > 5\}$
2. $\{x : x < 5\}$
3. $\left\{x : x < \frac{7}{3}\right\}$
4. $\{x : x \leq 10\}$
5. $\{x : x > -3\}$

Exercise 3 *(Refer to page 173 of Learner's Book 8)*

1. $\{x : x < 3\}$
2. $\{x : x > -2\}$
3. $\{x : x < 1\}$
4. $\left\{x : x < \frac{1}{2}\right\}$
5. $\{x : x \geq 7\}$

STRAND/CHAPTER 3: GEOMETRY AND MEASUREMENT

SUB-STRAND/UNIT 1: SHAPE AND SPACE

Exercise 1 *(Refer to page 184 of Learner's Book 8)*

1. $x = 106^\circ, y = 106^\circ$

2. a. 58° b. 48° c. 74° d. 122°

Exercise 2 *(Refer to page 186 of Learner's Book 8)*

a. $x = \frac{111}{8}$ *or* 13.875

Exercise 3 *(Refer to page 188-189 of Learner's Book 8)*

1. a. 1260°
   b. 1080°
2. $x = 104^\circ$
3. $x = 180^\circ$

STRAND/CHAPTER 3: GEOMETRY AND MEASUREMENT
SUB-STRAND/UNIT 2: MEASUREMENTS

Exercise 1 *(Refer to page 209 of Learner's Book 8)*

1. $1386cm^2$
2. $3,850cm^2$
3. $22cm^2$
4. $44cm^2$
5. $132cm^2$

Exercise 2 *(Refer to page 213 of Learner's Book 8)*

1.
   i. $\sqrt{29}\,cm^2$
   ii. $\sqrt{105}\,cm^2$
   iii. $\sqrt{95}\,cm^2$

2. $x = 7cm$
   $h = \sqrt{120}\,cm^2 = 2\sqrt{30}\,cm^2$
3. $p = 4$

Exercise 3 *(Refer to page 216-217 of Learner's Book 8)*

1. $2\sqrt{5}\,cm$
2. $t = 4cm$
   $y = \sqrt{52}\,cm$
3. $y = \sqrt{104}\,cm$
   $x = 4cm$
   $w = \sqrt{136}\,cm$

Exercise 4 *(Refer to page 218 of Learner's Book 8)*

1. a. Length of ladder $= 14.3m$
2. i. Height of triangle $= 10.39cm$
   ii. Area of triangle $= 62.35cm^2$
   iii. Perimeter of triangle $= 36cm$

Exercise 5 *(Refer to page 221 of Learner's Book 8)*

i. $\sin x = \frac{12}{13}$

ii. $\cos x = \frac{5}{13}$

iii. $\tan x = \frac{12}{5}$

iv. $\frac{12}{5} = \frac{12}{13} \times \frac{13}{5}$

Exercise 6 *(Refer to page 224 of Learner's Book 8)*

1. $31m$
2. Angle of depression are β, γ
   Angle of elevation are α, θ
   ii. $x = 14.5cm,\ y = 7.5cm,\ z = 13.6cm$
   iii. $\sin\alpha = \frac{7.5}{14.5}, \cos\theta = \frac{14.5}{15}, \tan\beta = \frac{7.5}{13}, \sin\gamma = \frac{4}{13.6}$

Exercise 7 *(Refer to page 235 of Learner's Book 8)*

1. i. $\begin{pmatrix} 14 \\ 5 \end{pmatrix}$ ii. $\begin{pmatrix} 6 \\ -7 \end{pmatrix}$ iii. $\begin{pmatrix} 11 \\ -3 \end{pmatrix}$
2. $x = 3$
   $y = -2$

STRAND/CHAPTER 4: HANDLING DATA

SUB-STRAND/UNIT 1: DATA

Exercise 3 *(Refer to page 246 of Learner's Book 8)*

Length (mm)	Tally	Frequency
25-29	//	2
30-34	////	4
35-39	~~////~~ //	7
40-44	~~////~~ ////	9
45-49	~~////~~///	8
50-54	~~////~~//	7
55-59	///	3

Exercise 4 *(Refer to page 248 of Learner's Book 8)*

$y = 40$ teachers.
University graduates - 1440 Diplomats- 1080
Specialist - 600
Others - 480

Exercise 5 *(Refer to pages 249 and 250 of Learner's Book 8)*

1. $B7 = 31\%$
   $B8 = 67\%$
   $B9 = 82\%$
   $B10 = 54\%$

Exercise 6 *(Refer to page 252 of Learner's Book 8)*

1.

Stem	leaf
1	5
2	0 0 4 8 9
3	9
4	8 8 9
5	6 6 6 9 9
6	
7	1 2 3 3 4

2. i. 10, 14, 16, 19, 19, 30, 30, 68, 68, 69, 73, 73, 74, 76, 98
   ii. mode: 19; 30; 68; 73
   iii. median 68

Exercise 7 *(Refer to page 259 of Learner's Book 8)*

1. Mean 4.65 median 5.5 mode 6
2. b. mean 3.04 ii. mode 2 iii. median 3
   c. The value are different because the numbers that appeared after the die was tossed were not normally distributed. thus the numbers did not come close enough to the mean value.

STRAND/CHAPTER 4: HANDLING DATA
SUBSTRAND/UNIT 2: PROBABILITY OR CHANCE

Exercise 2 *(Refer to page 266 of Learner's Book 8)*